Mean First Passage Time and Escape Rate of Improved Predator-Prey Model with Color-Correlated Gaussian Noise

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Abstract: Changing the initial state of a population has been proven to have profound impacts on the dynamics of the system and the long-term sustainability of species. Relevant studies have revealed the sensitivity of system parameters, providing a new perspective on system dynamics. The steady-state probability distribution function, mean first passage time and escape rate are obtained by theoretical calculation to discuss the effects of system parameters on the statistical properties of the improved predator-prey model. Obtained results show that the peak of the steady-state probability distribution function decreases with the increase of the multiplicative noise intensity, and its peak decreases with the increase of the birth rate related parameter, and the peak gradually shifts to the right. However, the peak of steady-state probability distribution function gradually becomes larger and more obvious when the correlation strength between noise increases. The mean first passage time $T_+(x_-\to x_u)$ and $T_-(x_+\to x_u)$ both show a trend of increasing first and then decreasing with the increase of additive noise intensity, but $T_-(x_+\to x_u)$ changes faster. With the increase of multiplicative noise intensity, $T_+(x_-\to x_u)$ shows a non-monotonic behavior of rapid rise and slow decline, while $T_-(x_+\to x_u)$ shows a trend of rising first and then declining, with roughly the same speed of rising and falling. The escape rate $W(x_\pm\to x_u)$ shows a monotonically decreasing trend with the noise correlation strength, while its function image with the noise intensity shows a monotonically increasing trend. Above results can provide some theoretical guidance for the protection of endangered species from the perspective of statistical physics to ensure that biodiversity conservation is carried out more effectively.

Keywords: Color Correlation Noise; Predator and Prey; Environmental Effect; Mean First Passage Time; Escape Rate

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1 Introduction

At the beginning of the 20th century, Lotka and Volterra established the predator-prey model, which is of great significance to the protection of ecological environment. Therefore, the study of this model has been a hot topic in ecology [1]. Any biological population is in a colony and has an inseparable connection with other populations. In the predator-prey model, predators and prey are both interdependent and interdependent [2]. In the ecosystem, the dynamic balance between predator and prey is the key to system stability. However, in reality, many influencing factors are accompanied by significant random fluctuations, and the nonlinear interference behavior to the sys-

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tem has become a hot topic of interdisciplinary research, which has been widely concerned by researchers in ecology and mathematics [3].

This kind of model has intuitive significance in describing the relationship between predator and prey, but its internal nonlinear influence mechanism makes the establishment of theory and numerical analysis face multiple challenges. For example, sudden environmental changes can indirectly trigger a chain reaction in the system by changing the efficiency of predators, which can lead to a vicious circle [4]. Studies have shown that when the fertility rate of predators is consistently lower than the mortality rate, predators tend to become extinct. It will also affect predators through the food chain and produce famine effect, leading to the extinction of both predators and prey. To prevent this from happening, the significance of studying predator and prey models is obvious [5].

Many researchers have discussed the problem of mean first passage time under the influence of noise in predator-prey models [6-8], but most researchers discuss interactions in the form of white correlation. In practice, it is more valuable to discuss colored correlation. Based on the existing predator-prey model, additive noise and multiplicative noise under the form of colored correlation are introduced to analyze two unstable points and one stable point of the system, and discuss the changes of the statistical properties of the system under different conditions.

2 Models and Methods

In the absence of predators, the population dynamics of prey species are determined by natural growth and migration [9]. Natural growth follows the Logistic growth model, ensuring that predator populations tend toward environmental carrying capacity [10]. Migration, on the other hand, has an additional effect on population size through individual mobility. In addition, in the predator and prey model, the survival of the predator depends entirely on the prey [11]. If there is no prey, the predator will go extinct because of lack of food.

2.1 Derivation of Langevin Equation

The activity state of predators can be divided into two simple states, i.e., predator state and rest state. The time when the predator is in the state of predation is denoted by $_R$, and is the survival time of the prey [12]. And $_H$, $R <<$ (is the birth rate of predators). Thus a differential relationship between predator and prey population can be obtained:

$$\frac{dX}{dt} = A + \mu X(1 - \frac{X}{K}) - \frac{1}{\tau_H} XY. \quad (1)$$

$$\frac{dY}{dt} = -\frac{1}{\tau_H} XY + \frac{1}{\tau_R} Z. \quad (2)$$

Here, X represents the number of predators, Y and Z represent the number of predators in the predator state and the number of predators in the rest state respectively, and A represents the number of predators that enter the field through migration [13]. $X(1\text{-}X/K)$ indicates that the number of prey increases in a Logistic growth mode without predators, where K refers to the maximum number of prey that the environment can support [14]. If one use the constant E to represent the total number of predators, you have a constant $E = Y(t) + Z(t) = \text{Constant}$.

Eqs. (1) and (2) take into account the actual natural conditions and the mixed model of Logistic model. The parameters of above equations are too many to facilitate analysis [15]. In order to study the overall trend and dynamics of predator and prey population in more detail over time , the corresponding transformation is made as follows [16]:

$$\tau_H = \varepsilon\tau_H^*, \tau_R = \varepsilon\tau_R^*, Y = \varepsilon Y^*, Z = \varepsilon Z^*. \quad (3)$$

In above equations, ε is a small quantity. Substituting Eq. (3) into Eqs. (1) and (2), we can get [17]:

$$\frac{dX}{dt} = A + \mu X(1 - \frac{X}{K}) - \frac{1}{\tau_H^*} XY^*. \quad (4)$$

$$\varepsilon\frac{dY}{dt} = -\frac{1}{\tau_H^*} XY^* + \frac{1}{\tau_R^*} Z^*. \quad (5)$$

Eliminate Y^* from Eq. (4), and consider the extreme case, there is a limit $\rightarrow$. After the transformation, a dimensionless differential equation about the evolution of predators over time can be obtained [18]:

$$\frac{dx}{dt} = \alpha + x(1 - \theta x) - \beta\frac{x}{1 + x}. \quad (6)$$

where

$$x = \frac{\tau_R^*}{\tau_H^*} X, \quad \alpha = \frac{A\tau_R^*}{\mu\tau_H^*}, \quad \beta = \frac{E}{\mu\tau_H}, \quad \theta = \frac{\tau_H}{\tau_R K}. \quad (7)$$

From Eq. (7), it can be seen that and are all pa-

rameters related to the birth rate of the prey [19]. Taking into account the randomness of individual genetic, behavioral, predator year differences and predator ability differences, as well as the effect of environmental fluctuations on predator birth rates, thus both and β are affected [20]. According to the different forms and classifications of noise, these random fluctuations are added to the system in the form of additive noise and multiplicative noise respectively, so that the stochastic differential equation of predator-prey model under the influence of noise can be obtained [21]:

$$\frac{dx}{dt} = \alpha + x(1-\theta x)\text{-}\beta\frac{\mathrm{x}}{1+x} - \frac{\mathrm{x}}{1+x}\xi(t) + \eta(t) \tag{8}$$

where, $\xi(t)$ and $\eta(t)$ are Gaussian colored noise, and the former is multiplicative noise, the latter is additive noise [22]. Their statistical properties are as follows:

$$<\xi(t)>=<\eta(t)>=0 \tag{9}$$

Their self-correlation function takes the form as follows:

$$<\xi(t)\xi(t')>=\frac{D}{\tau_1}\exp(\frac{-|t-t'|}{\tau_1}) \tag{10}$$

$$<\eta(t)\eta(t')>=\frac{Q}{\tau_2}\exp(\frac{-|t-t'|}{\tau_2}) \tag{11}$$

where, D and Q represents the intensity of multiplicative noise and additive noise respectively [23]. Since the discussion of Gaussian noise is in the form of colored correlation, the correlation expression between multiplicative noise and additive noise is as follows:

$$<\xi(t)\eta(t')>=<\eta(t)\xi(t')>=\frac{\lambda\sqrt{DQ}}{\tau_3}\exp(\frac{-|t-t'|}{\tau_3}) \tag{12}$$

where, the time between the inter-relatedness of the noise is denoted by , and the intensity of the inter-relatedness between the noise is denoted by , and meet the constraints -1< <1 [24].

2.2 Potential Function and Steady-State Solution

Assume that over a period of time, there are no predators migrating from other areas to the area under study, i.e. the number of predators migrating from other areas to the area is zero. At the same time, appropriate parameters are selected according to the relationship between predator and prey in reality [25]. Therefore, we can get one unstable point x_u and two stable points x_+ and x_- of the system through calculation [26]. Then, the potential function in the predator-prey model can be written in the following form:

$$V(x) = -\alpha x - \frac{1}{2}x^2 + \frac{\theta}{3}x^3 + \beta x - \beta\ln(x+1) \tag{13}$$

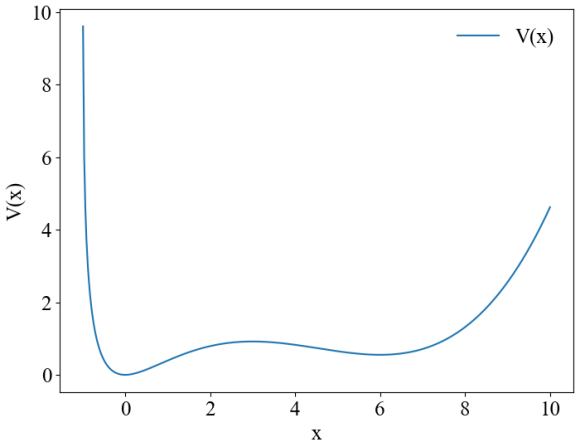

Figure 1 Graph of the deterministic potential function $V(x)$ at $\alpha = 0$, $\beta = 2.8$ and $\theta = 0.1$

According to Eq. (13), the deterministic potential function is drawn as shown in Figure 1. As can be seen from Figure 1, the potential function of the system has one unstable point and two stable points [27]. According to the results of numerical calculation, we can get [28]:

$$x_+ = \frac{-(\theta-1)+\sqrt{(\theta-1)^2-4\theta(\beta-1)}}{2\theta} \tag{14}$$

$$x_- = \frac{-(\theta-1)-\sqrt{(\theta-1)^2-4\theta(\beta-1)}}{2\theta} \tag{15}$$

According to the differential equation (6) between predator and prey population evolution, the following transformation is carried out [29]:

$$f(x) = \alpha + x(1-\theta x) - \beta\frac{x}{1+x} \tag{16}$$

$$g_1(x) = -\frac{x}{1+x} \tag{17}$$

$$g_2(x) = 1 \tag{18}$$

Then, the Langvin equation (8) under the effects of colored-correlated Gaussian noise can be transformed as

follows [30]:

$$\frac{dx}{dt} = f(x) + g_1(x)\xi(t) + g_2(x)\eta(t). \quad (19)$$

According to the relevant knowledge of Liouville equation, it can be obtained as follows [31]:

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial}[f(x) + g_1(x)\xi(t) + g_2(x)\eta(t)]P(x,t). \quad (20)$$

Using the knowledge of nonlinear kinetic theory and the corresponding Liouville equation, it can be known through Van-Kampen Lemma that the ensemble average is written as follows [32]:

$$P(x,t) =< \delta[x(t) - x] > . \quad (21)$$

Then the mean value of both sides of the above equation is obtained as follows [33]:

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial} f(x)P(x,t) - \frac{\partial}{\partial x} g_1(x) < \xi(t)\delta(x(t) - x) > - \frac{\partial}{\partial x} g_2(x) < \eta(t)\delta(x(t) - x) > . \quad (22)$$

According to Novikov's theorem, the above formula can be rearranged as follows:

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x} f(x)P(x,t) - \frac{\partial}{\partial x} g_1(x) < \xi(t)P(x,t) > - \frac{\partial}{\partial x} g_2(x) < \eta(t)P(x,t) > . \quad (23)$$

After further deformation, it is calculated as follows [34]:

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x} A(x)P(x,t) \frac{\partial^2}{\partial x^2} B(x)P(x,t). \quad (24)$$

In the above formula, $A(x)$ and $B(x)$ are written as follows [35]:

$$A(x) = f(x) + \frac{\mathrm{D}}{1-\tau_1 f^{'}(x_s)} g_1(x) g_1^{'}(x) + \frac{\lambda\sqrt{DQ}}{1-\tau_3 f^{'}(x_s)} [g_1(x) g_2^{'}(x) + g_1^{'}(x) g_2(x)] + \frac{Q}{1-\tau_2 f^{'}(x_s)} g_2(x) g_2^{'}(x) \quad (25)$$

$$B(x) = \frac{\mathrm{D}}{1-\tau_1 f^{'}(x_s)} g_1^2(x) + \frac{2\lambda\sqrt{DQ}}{1-\tau_3 f^{'}(x_s)} g_1(x) g_2(x) + \frac{Q}{1-\tau_2 f^{'}(x_s)} g_2^2(x) \quad (26)$$

$$P_{st}(x) = \frac{N}{B(x)} \exp\{\int \frac{f(x)}{B(x)} dx\} \quad (27)$$

The above expression is the analytical solution of the system under steady-state conditions, which is denoted as the steady-state probability distribution function $P_{st}(x)$ [36].

2.3 Mean First Passage Time and Escape Rate

In predator-prey models, the mean first passage time usually refers to the average time it takes for the system to reach a specific state for the first time from an initial state [37]. The traditional predator-prey model is deterministic, and the number of predators and prey will continue to oscillate, which cannot explain the extinction phenomenon in the natural environment. In the actual predator and prey model, environmental noise (such as resource fluctuation and natural disasters) and system randomness (the discreteness of individual birth and death) will affect the stability of the system, and even lead to the occurrence of extinction. The mean first passage time can be used to study the expected time when the number of predators and prey drops to zero for the first time, revealing the global stability of the system. Sensitivity changes can serve as an early warning signal of ecosystem collapse when the system approaches a bifurcation point.

Using the steepest descent method, the formula of the mean first passage time from two stable states to the unstable state can be obtained by numerical calculation:

$$T_+(x_- \to x_u) = \int_{x_-}^{x_u} \frac{dx}{B(x)P_{st}(x)} \int_{-\infty}^{x} P_{st}(y)dy, = \frac{2\pi}{\sqrt{|V''(x_u)V''(x_-)|}} \exp[U(x_u) - U(x_-)] \quad (28)$$

$$T_-(x_+ \to x_u) = \int_{x_+}^{x_u} \frac{dx}{B(x)P_{st}(x)} \int_{-\infty}^{x} P_{st}(y)dy = \frac{2\pi}{\sqrt{|V''(x_u)V''(x_+)|}} \exp[U(x_u) - U(x_+)]. \quad (29)$$

In the predator-prey model, escape rate refers to the probability of prey escaping when it meets the predator. Its significance is mainly reflected in the influence on the hunting efficiency and the regulation of the dynamic relationship between predator and prey. A higher escape rate will directly affect the predator's hunting efficiency, and the predator may suffer from a population shrinkage due

to lack of food. In the long run, escape rates may promote the evolution of more efficient escape strategies in predators, and predators may develop more efficient capture strategies. Therefore, escape rate can directly affect the dynamic relationship between predator and prey by adjusting the efficiency of predation.

The escape rate can be calculated using the following equation [9]:

$$W_{\pm} = \frac{\sqrt{|V''(x_u)V''(x_{\pm})|}}{2\pi}\exp[U(x_{\pm}) - U(x_u)]. \quad (30)$$

3 Results and Discussion

According to Eq. (21), the steady-state probability density distribution function $P_{st}(x)$ with respect to x can be drawn, as shown in Figure 2(a) and (b). In Figure 2(a), the value of parameter α is chosen as 0.05, and the graph shows the image of $P_{st}(x)$ as a function of x under different multiplicative noise intensities D. As can be seen from Figure 2(a), with the decrease of noise intensity D, the peak value of $P_{st}(x)$ decreases, so the decrease of multiplicative noise intensity D reduces the stability of the system. However, as the noise intensity D changes, the position of the peak remains almost unchanged. From the perspective of predator populations, it means that an increase in the intensity of multiplicative white noise will lead to an enhancement in the stability of the prey population.

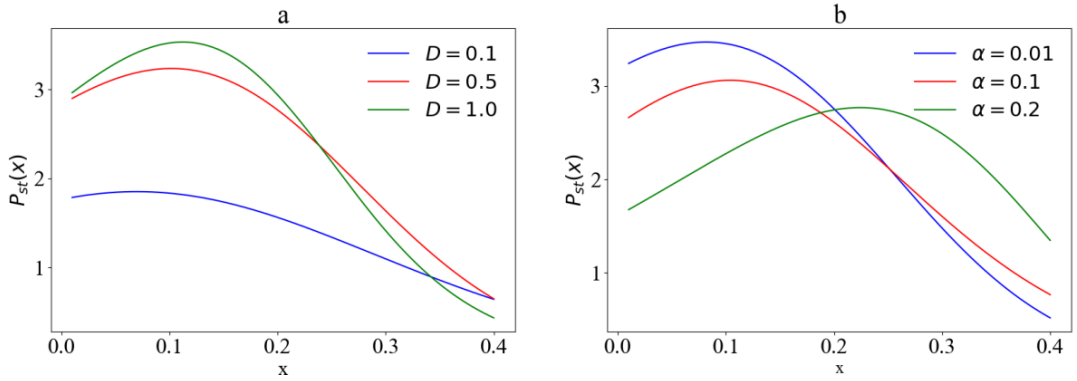

Figure 2 (a) The variation of steady-state probability density distribution function $P_{st}(x)$ as a function of x at different multiplicative noise intensity D at α = 0.05, Q = 0.1, β = 2.8, θ = 0.1, τ_1 = 0.1, τ_2 = 0.3, τ_3 = 0.5 and λ = 0.9; (b) The variation of steady-state probability density distribution function $P_{st}(x)$ as a function of x at different α at D = 0.5, Q = 0.1, β = 2.8, θ = 0.1, τ_1 = 0.1, τ_2 = 0.3, τ_3 = 0.5 and λ = 0.9

In Figure 2(b), if the value of multiplicative noise intensity is fixed to D = 0.5, it can be seen that with the decrease of parameter , the peak of the system increases continuously, and the position of the peak moves to the left gradually. When parameter α decreases to a certain value, the peak of the steady-state probability density distribution function gradually disappears and the curve becomes a monotonic decreasing curve. These results mean that the system is on the verge of extinction, and that the system is shifting from a stable state to an unstable state.

Further consider the influence of the correlation strength between noises ($\lambda \neq 0$) on the steady-state probability density distribution function, and make a graph of $P_{st}(x)$ about x according to Eq. (16), as shown in Figure 3. As can be seen from Figure 3, with the increase of the correlation strength λ between the noises, the peak of the probability density distribution function becomes higher and more obvious, indicating that the stability of the system is enhanced. When the value of the correlation strength between noise is chosen as , the steady state probability density distribution function does not show a peak, indicating that there is no steady state when the given noise is negative correlation. Moreover, as the correlation strength between noise λ increases, the system becomes more stable. Therefore, it can be inferred that when the correlation strength between noises meets the condition -1 < < , the system will be in a more unstable state and even extinct.

It can also be seen from Figure 3 that when the value of noise inter-correlation strength is λ = 1, the steady-state probability density distribution function decreases the fastest, and the system tends to collapse, which indicates that there are two sides of noise inter-correlation strength. Therefore, in the actual ecosystem, the inter-relatedness of noise should be kept at a moderate level to ensure that the stability of the system is not easy to lead to extinction.

These results indicate that when the noise is negatively correlated, the peak of the steady-state probability distribution function of the system disappears, indicating that the system may undergo a phase transition, which could be fatal to predator populations.

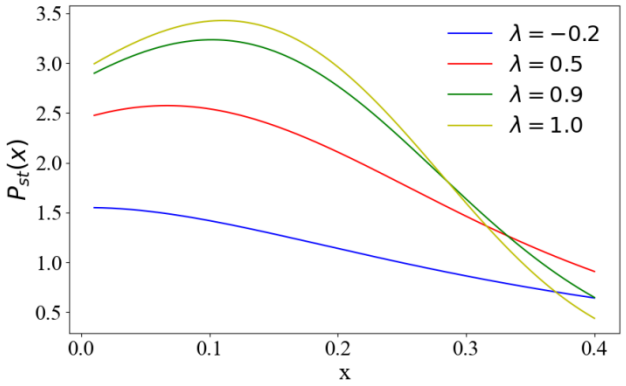

Figure 3 The variation of steady-state probability density distribution function $P_{st}(x)$ as a function of x under different noise correlation strength λ at $D = 0.5$, $Q = 0.1$, $\alpha = 0.05$, $\beta = 2.8$, $\theta = 0.1$, $\tau_1 = 0.1$, $\tau_2 = 0.3$ and $\tau_3 = 0.5$

Figure 4 shows the mean extinction time $T(x_{\pm} \rightarrow x_u)$ as a function of additive noise intensity Q corresponding to different multiplicative noise intensities D. According to the expression of mean first passage time, when the predator changes from the stable state x_+ to the unstable state x_u, the graph of its mean extinction time $T_+(x_- \rightarrow x_u)$ about the noise intensity Q is shown in Figure 4(a). It can be seen from the image change in Figure 4(a) that with the change of additive noise intensity, the average extinction time $T_+(x_- \rightarrow x_u)$ of the system shows a non-monotonic change trend of rapid rise and slow decline, which means that the additive noise intensity Q has a critical threshold effect on the extinction time of the prey.

With the increase of additive noise intensity Q, the mean extinction time of predators will reach a certain threshold and then decrease at a slow rate, which indicates that environmental fluctuations or individual behavior differences can significantly prolong the mean extinction time of predators, thus enhancing the stability of the population. In addition, with the increase of multiplicative noise intensity D, the height of the resonance peak of steady-state probability density distribution function gradually decreases, and the Q value corresponding to the peak moves slightly to the right, indicating that the strong additive noise intensity Q has a weakening effect on the mean first passage time.

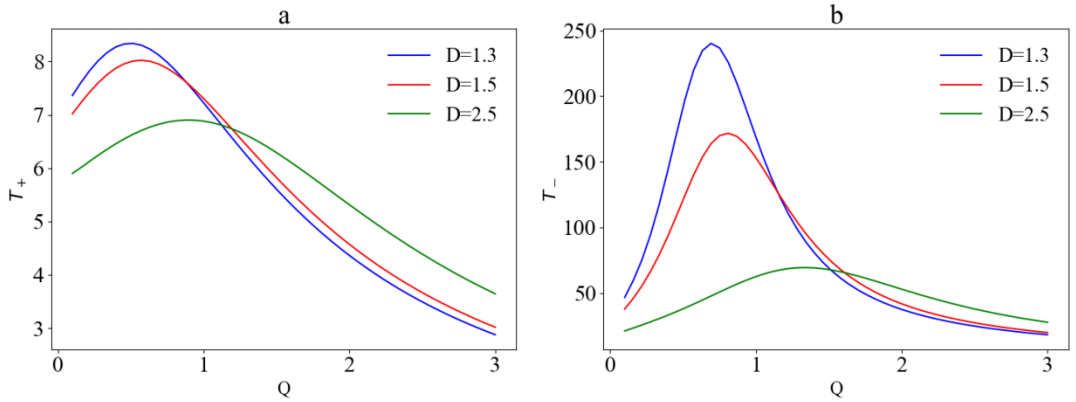

Figure 4 Mean extinction time $T(x_{\pm} \rightarrow x_u)$ as a function of additive noise intensity Q under different multiplicative noise intensity D at $\alpha = 0$, $\beta = 2.8$, $\theta = 0.1$, $\tau_1 = 0.1$, $\tau_2 = 0.1$, $\tau_3 = 0.1$ and $\lambda = 0.9$. (a) $T_+(x_- \rightarrow x_u)$; (b) $T_-(x_+ \rightarrow x_u)$

As can be seen from Figure 4(b), when the number of predators changes from the stable state x_- to the unstable state x_u, the mean extinction time $T_-(x_+ \rightarrow x_u)$ as a function of additive noise intensity Q shows a non-monotonicity of first rising and then decreasing. From the perspective of comparison, the overall trend of the change of its image is similar to that in Figure 2(a), but the mean extinction time is generally shorter. The image curves in Figure 4(a) and Figure 4(b) also show the characteristics of peak decline and rightward shift, but the difference is that the peak attenuation amplitude in Figure 4(b) is more significant. This difference suggests that the process of evolution to extinction is asymmetric when predators are exposed to the same external stimuli but in two different stable states.

The two graphs in Figure 5 show the mean extinction time $T(x_{\pm} \rightarrow x_u)$ as a function of multiplicative noise intensity D with additive noise intensity Q. In Figure 5, the influence of multiplicative noise intensity D on mean ex-

tinction time $T(x_{\pm}\rightarrow x_u)$ is constrained by additive noise intensity Q, indicating that the noise has a "double-edged sword" effect, and the correlation mechanism between noises plays an important role in regulating the mean extinction time. It can be seen from the change trend of the two images in Figure 5 that when the additive noise Q is fixed, with the increase of the multiplicative noise intensity D, $T_{+}(x_{-}\rightarrow x_u)$ presents a non-monotonic behavior of first rising and then decreasing, which indicates that moderate multiplicative noise can prolong the extinction time of the prey. However, if the multiplicative noise intensity D is too strong, its stability will be destroyed. In addition, with the increase of additive noise intensity Q, the height of resonance peak gradually decreases, and the peak position shifts to a larger multiplicative noise intensity D, which means that there is a synergistic effect between additive noise and multiplicative noise.

It can be seen from the comparison of Figure 5(a) and Figure 5(b) that the overall trend of the image change is similar, but there are some differences. At the same additive noise intensity Q value, the peak height in Figure 5(a) may be lower, and its peak position moves toward the direction with larger multiplicative noise intensity. If the multiplicative noise intensity Q decreases, the mean extinction time in Figure 5(b) is more sensitive to the multiplicative noise intensity Q, which indicates that the system has asymmetry in response to noise when the prey is in different stable states. At the same time, it is worth noting that there are some differences between this and the previous research results, i.e., the appearance of non-monotonic function relationship suggests that the interconnection form of noise is a key factor affecting the dynamics of the system. These results show that the interrelated form of noise also has a certain impact on the system. Multiplicative noise and additive noise jointly regulate the extinction time of prey, and there is an optimal noise combination that maximizes the extinction time, which provides a theoretical basis for population protection strategies.

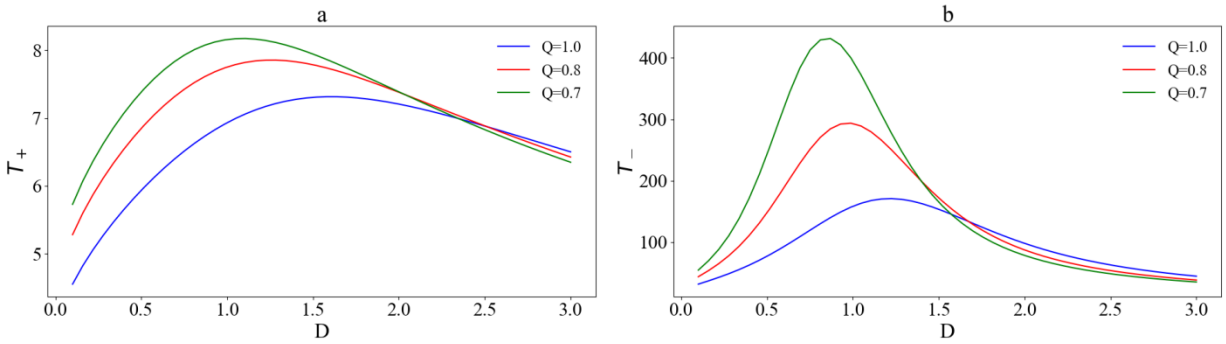

Figure 5 Mean extinction time $T(x_{\pm}\rightarrow x_u)$ as a function of multiplicative noise intensity D under different additive noise intensity Q at $\alpha = 0$, $\beta = 2.8$, $\theta = 0.1$, $\tau_1 = 0.1$, $\tau_2 = 0.1$, $\tau_3 = 0.1$ and $\lambda = 0.9$. (a) $T_{+}(x_{-}\rightarrow x_u)$; (b) $T\text{-}(x_{+}\rightarrow x_u)$

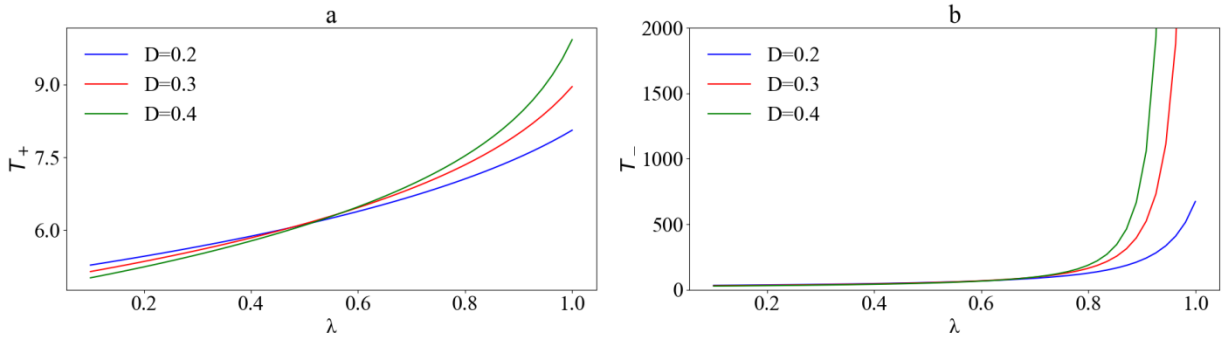

Figure 6 Mean extinction time $T(x_{\pm}\rightarrow xu)$ as a function of noise correlation strength λ under different multiplicative noise intensity D at $\alpha = 0$, $Q = 0.1$, $\beta = 2.8$, $\theta = 0.1$, $\tau_1 = 0.1$, $\tau_2 = 0.1$, $\tau_3 = 0.1$. (a) $T_{+}(x_{-}\rightarrow xu)$; (b) $T_{-}(x_{+}\rightarrow x_u)$

Figure 6(a) and (b) show the images of $T(x_{\pm}\rightarrow x_u)$ as a function of noise correlation strength λ with the change of the multiplicative noise intensity D. As can be seen from Figure 6, when the given initial conditions are different, that is, if the predator starts at two different stable states x_{-} and x_{+}, then the noise correlation strength λ as different

effects on the mean first passage time $T(x_{\pm}\rightarrow x_u)$.

From the analysis of Figure 6(a), it can be found that if the predator is in the stable state x_-, the mean extinction time $T_+(x_-\rightarrow x_u)$ increases monotonically with the noise correlation strength λ for all values of multiplicative noise intensity D. That is, when the correlation strength between noise increases, the mean first passage time from one state to another for the predator becomes longer. When the value range of the correlation strength between noise λ is between 0.4 and 0.6, the mean extinction time corresponding to different multiplicative noise intensity D is close to each other, indicating that different multiplicative noise intensity D has little influence on the mean first passage time $T_+(x_-\rightarrow x_u)$ within this range. When the value of correlation strength between noise λ is greater than 0.6, the difference between $T_+(x_-\rightarrow x_u)$ becomes more obvious with the increase of multiplicative noise intensity D, indicating that the positive correlation effect of external noise can significantly prolong the extinction time of prey and enhance the stability of the system.

When the initial condition of the predator is in the stable state of x_+, the time evolution of the mean first passage time $T_-(x_-\rightarrow x_u)$ with the noise correlation strength λ is shown in Figure 6(b). It can be seen from the analysis of Figure 6(b) that $T_-(x_-\rightarrow x_u)$ is an increasing function of λ, and the mean extinction time will increase with the increase of the correlation strength λ. These results suggest that the random fluctuations of the external environment are beneficial to the predator, which leads to the enhancement of the stability of the system.

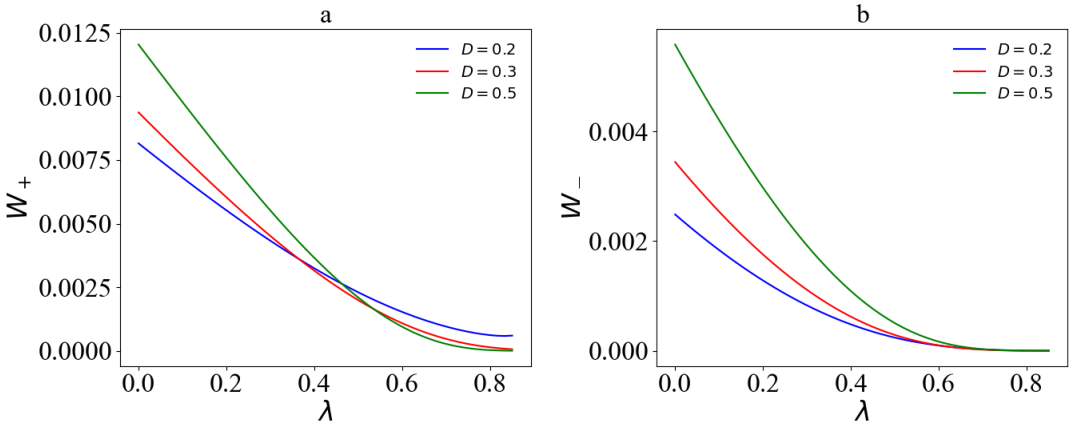

Figure 7 Escape rate $W(x_{\pm}\rightarrow x_u)$ as a function of noise correlation strength λ under different multiplicative noise intensity D at $\alpha = 0$, $\beta = 2.8$, $\theta = 0.1$, $\tau_1 = 0.01$, $\tau_2 = 0.01$, $\tau_3 = 0.01$, $Q = 0.2$

If the value of correlation strength between noise λ is close to 0, the influence of different multiplicative noise intensity D on the mean extinction time $T_-(x_-\rightarrow x_u)$ is negligible, and the system is in a relatively stable state. When the correlation strength between noise λ increases, the average extinction time of prey increases more with the increase of multiplicative noise intensity D, and the improvement of system stability is more significant. It shows that an appropriate increase in noise intensity helps to maintain the population size and structural stability of predators in the ecosystem. It suggests that fluctuations caused by genetic inheritance, environment, individual behavior habits and other factors can promote the evolution of the population when the correlation intensity between noise is increased, and promote the whole population to evolve towards a more adaptive direction.

According to the escape rate in Eq. (19), the graph of the system escape rate as a function of the noise correlation strength λ under different multiplicative noise intensity D is shown in Figure 7. In Figure 7(a), the escape rate W_+ shows an overall monotonic decreasing trend, that is, with the increase of noise correlation strength λ, the escape rate gradually decreases to 0. In addition, with the decrease of multiplicative noise intensity D, the escape rate decreases faster. In previous studies [11], researchers have focused on the impact of positive and negative correlations on the transient properties of populations, while rarely considering the different states of prey populations. Our research indicates that the impact of noise on escape rate varies when the prey is in different states.

As can be seen from Figure 7(b), the overall trend of escape rate W_- is similar to that in Figure 7(a). The difference is that the escape rate decreases rapidly to 0 with the increase of noise correlation strength λ, and the escape rate decreases faster with the decrease of multiplicative noise intensity D. Above results show that the increase of

λ hinders the transition from meta-stable state to stable state and reduces the escape rate of population.

Taking the noise correlation strength λ as the variable and changing the additive noise intensity Q, the variation trend of the system escape rate is shown in Figure 8. As can be seen from Figure 8(a), when the additive noise intensity Q is selected as Q = 0.1, the escape rate W_+ shows a significant downward trend with a rapid decline. When the value of additive noise intensity is chosen as Q = 0.05, the escape rate W_+ remains almost unchanged with the increase of noise correlation intensity λ. It is worth noting that when the value of noise correlation strength is selected as λ = 0.8, the escape rate W_+ is almost 0, indicating that random fluctuations have a great impact on the predator population in this case. These results indicate that an increase in the intensity of additive noise significantly affects the systematic escape rate of prey populations, leading to rapid extinction under specific conditions.

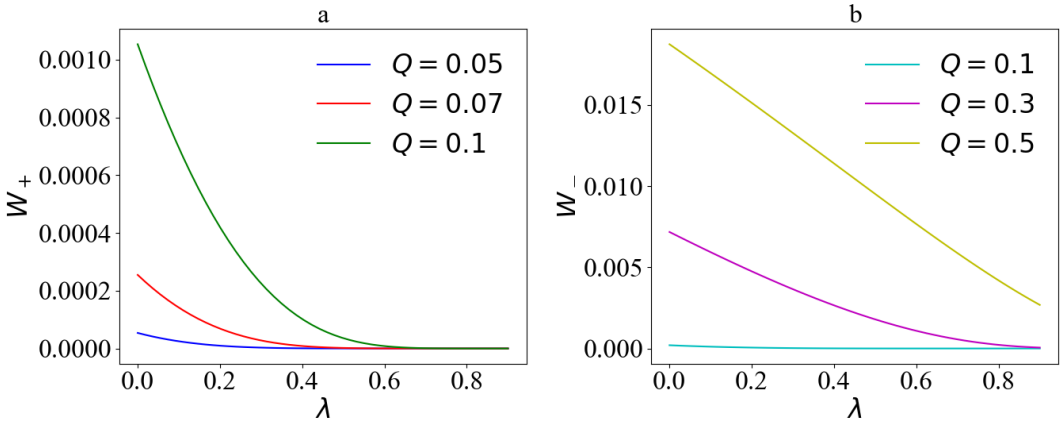

Figure 8 Escape rate $W(x_\pm \to x_u)$ as a function of noise correlation strength λ under different additive noise intensity Q at D = 0.5, α = 0, β = 2.8, θ = 0.1, τ_1 = 0.01, τ_2 = 0.01 and τ_3 = 0.01. (a) $W_+(x_- \to x_u)$; (b) $W_-(x_+ \to x_u)$

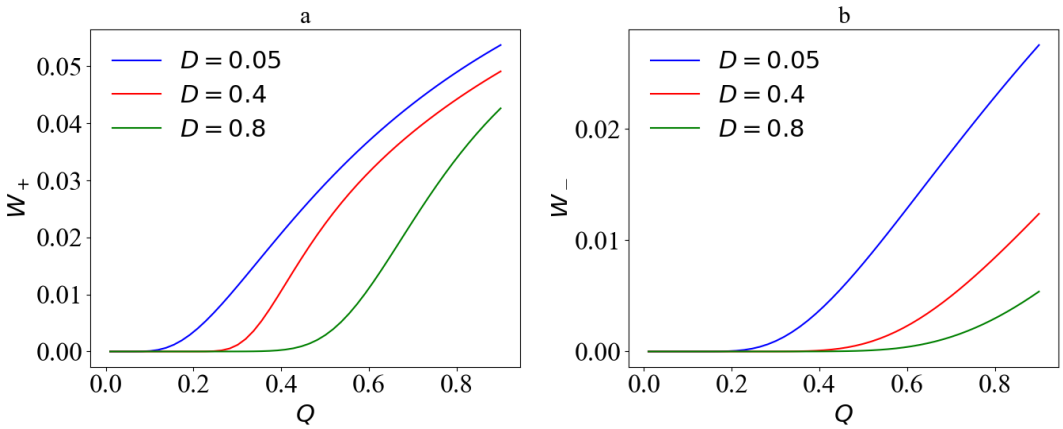

Figure 9 Escape rate $W(x_\pm \to x_u)$ as a function of additive noise intensity Q under different multiplicative noise intensity D at α = 0, β = 2.8, θ = 0.1, τ_1 = 0.01, τ_2 = 0.01, τ_3 = 0.01 and λ = 0.9. (*a*) $W_+(x_- \to x_u)$; (b) $W_-(x_+ \to x_u)$

As can be seen from Figure 8(b), when the value of additive noise intensity is set as Q = 0.1, the escape rate W_- remains almost unchanged with the change of noise correlation intensity λ. However, when the value of additive noise intensity is Q = 0.5, the escape rate W_- and the correlation strength between noise λ show a linear correlation, which is beneficial to the study of ecological environment. In the real ecological environment, the population should be maintained under appropriate conditions to promote the development of the population. These results indicate that the increase in additive noise intensity has a consistent impact on the escape rate of prey systems when they are in another unstable state.

Taking the additive noise intensity Q as the variable and changing the multiplicative noise intensity D, the escape rate image of the system is shown in Figure 9. As can be seen from Figure 9(a), the escape rate shows an overall monotonic increasing trend. With the increase of additive noise intensity Q, the escape rate W_+ increases gradually, and the smaller the D is, the larger the value of

escape rate is.

As can be seen from Figure 9(b), the escape rate W_- also shows a monotonically increasing trend with the increase of noise intensity Q. When the value of multiplicative noise intensity is $D = 0.05$, the escape rate W_- increases significantly. Therefore, it can be concluded that the change of multiplicative noise intensity D has a relatively obvious effect on the escape rate, which may lead to system collapse and ecological environment destruction.

4 Conclusions

This article introduces colored correlated noise into the classical predator-prey model and solves for two stable points and one unstable point in the system. Using the system's mean first passage time and escape rate, the impact of correlated noise on the mean extinction time of prey and escape rate is systematically explored. The results show that the average extinction time of the system exhibits a non monotonic trend of first increasing and then decreasing with the change of noise intensity, and the position and magnitude of its peak are modulated by both multiplicative and additive noise. When the mean extinction time is a function of the noise correlation strength, it shows a monotonically increasing trend, which is significantly different from the impact of noise on it. The escape rate shows a monotonic decreasing trend as a function of the noise correlation strength, but it shows a monotonic increasing trend with the additive noise intensity, showing the different effects of noise intensity and noise correlation strength on the escape rate of the system. The study of predator-prey models holds profound interdisciplinary significance for maintaining ecosystem stability. Relevant research findings provide theoretical guidance for assessing the risk of species extinction under climate change. By constructing stochastic differential equations to simulate population fluctuations, it offers quantitative evidence for determining the minimum viable population size and designing ecological corridor widths as conservation measures.

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