Control System of Uncertain System with Model-Free Design Based on Newton's Laws of Motion

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Abstract: Design of control systems needs a mathematics model of controlled plant in general, however the model is not exactly obtained and the model is uncertain, these limits led to the poor control system performance. The paper aim at when a motion (/reaction) speed of controlled plant process (/body) is greatly slower than speed of light, its motion process can described by Newton's Laws of Motion, and these laws can be applied to the design and analysis the control systems quality. This paper designed 3 control systems: PID control system with improved Smith predictor in time-invariance systems, PIDC control system with compensator (in time-variance uncertain systems) and CSMFDNLM (Control System of Uncertain System with Model-Free Design Based on Newton's Laws of Motion). A control system for uncertain plan with Model-Free is designed based on Newton's laws of motion in the paper. An OUAM filter based on Uniform Acceleration Movement is proposed by applying Kalman filter theory with constructing three state variables of the controlled system, i.e., position, speed and acceleration. The simulation and application results of PIDC control system with compensator and CSMFDNLM showed good control quality and robust performance for uncertain systems.

Keywords: Newton’s Laws of Motion; Kalman Filter; Observer Based on Uniform Acceleration Movement; Model-Free Design Method; PID Controller

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1 Introduction

Newton's Laws of Motion are foundation of control theory and application, and are widely applied in the control projects. PID controller, PIDC system (PID Control with Compensator) and CSMFDNLM (Control System of Uncertain System With Model-free Design based on Newton's Laws of Motion) system are emphatically analyzed in the paper. The unbiased estimation of OUAM observer and effectiveness of CSMFCNLM system are demonstrated. The unity of classical mechanics principle are explained in these control systems.

2 PID Control System and PIDC System (PID Control with Compensator)

2.1 PID Control System

PID control system is often utilized in engineering practice because to its simplicity, stability, and ease of tuning.

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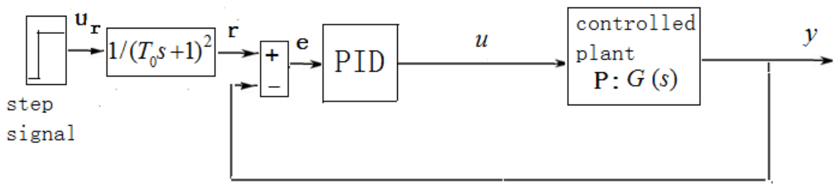

Figure 1 PID control system

In Figure 1, $PID(s)$ controller is designed, based on controlled plant $G(s)=ke^{-\tau s}/(as^2+bs+1)$,

$PID(s)=k_r(k_p+k_i/s+k_d s/(t_d s+1))$,

where, k_r, k_p, k_i, k_d and t_d , these parameters are optimized according to some optimization criterions [2, 5, 11, 13, 15] including:

$$\begin{aligned} &IAE: S=\int_0^\infty |e(t)|\,dt \\ &ISE: S=\int_0^\infty |e(t)|^2\,dt \\ &ITAE: S=\int_0^\infty t\,|e|\,dt \end{aligned} \tag{1}$$

u_r is a step signal with amplitude $|u_r|=r$, $1/(T_0 s+1)^2$ is a loop adjusted respond speed and amplitude of output y.

If the controlled plant $G(s)=ke^{-\tau s}/(as^2+bs+1)$ is a linear time-invariance system, the PID control system of Figure 1 exhibits good performance and control quality.

2.2 Improved Smith Predictor for the Pure Delay [2, 7, 11, 12]

Assume the controlled plant $G(s)=ke^{-\tau s}/(as^2+bs+1)=G_0(s)e^{(-s\tau)}$, where, $G_0(s)=k/(as^2+bs+1)$, pure delay $e^{(-s\tau)}$ is expressed as

$$e^{(-s\tau)}=1-s\tau+(s\tau)^2/2-(s\tau)^3/6+...$$

Improved Smith predictor is shown in Figure 2 and Figure 3, the design method is divided into 2 steps:

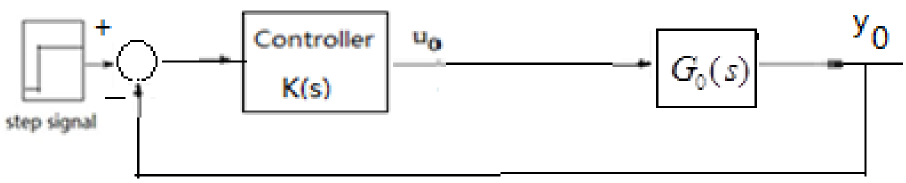

Figure 2 PID control system with improved Smith predictor (the first step design)

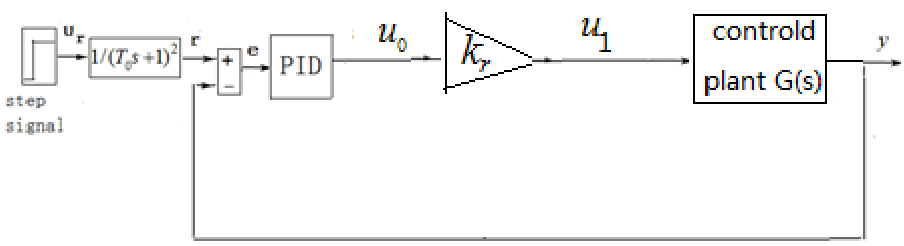

Figure 3 PID control system with improved Smith predictor (the second step design)

In Figure 2, controller K(s) is designed based on $G_0(s)$ according to some optimization criterions, the output y_0 of closed loop is shown in Figure 3. This is the first step design without pure delay loop.

In Figure 3, k_r is calculated, so that output y_0 *(in Figure 3)* is shifted time τ to y *(in Figure 4)* from zero toward to the right. In general, if $\tau=0$, then $k_r=1$; if $\tau/T_t\neq 0$, then $k_r\in(0\ 1)$.

In Figure 1,

$$\begin{aligned} &let\ m^{-1}=e^{\tau s}, \\ &G_0(s)=G(s)m^{-1}=G(s)e^{\tau s}=k/(as^2+bs+1) \end{aligned} \tag{2}$$

m is considered as the mass of a body in classical mechanics, $\tau=0$ in above Equation, $m=e^{-\tau s}$ (inertance), $m_{G_0(s)}=1$, so the body became a moving mass point.

y_0 (in Figure 3) is shifted time τ to y (in Figure 4) from zero toward to the right using calculating and adding k_r, the operation equivalents to the body becoming a moving mass ($m_{G(s)} = e^{-\tau s}$) from $m_{G_0(s)} = 1$ to $m_{G(s)} = e^{-\tau s}$ (inertance).

If $\tau/T_t \neq 0$, we always find some k_r so that output y_0 (in Figure 4) is shifted time τ to y (in Figure 5).

Let $\tau/T_t=X$, $k_r=f(X)=c_1X+c_2X^2+c_3X^3+c_0$; $k\to 1$, *as* $\tau\to 0$ and Tt=ccnstant; kr→0, as τ= ccnstant; where c1, c2, c3, and c0 are solved by polynomial regression algorithm.

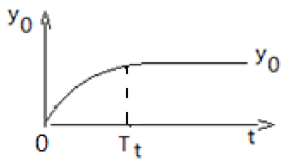

Figure 4 Output y_0 of closed loop based on $G_0(s)$

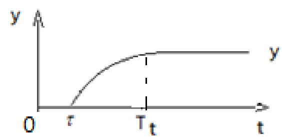

Figure 5 Output y of closed loop based on $G(s)$

2.3 PIDC Control Systems with Compensator [5, 7, 9, 10, 12]

PID Control systems operated at fine control quality index for liner time-invariant plants (certain models), however, the poor tuning and control quality index are found in the industrial application of PID Controller within time –variant plants/non-linear plants (uncertain models), because of its fixed structure and parameters.

A compensator is designed to enhance control quality and robust performance in time –variant plants (uncertain models) or non-linear plants in the paper, the *PID* controller is designed based on the transfer function of nominal operation case, under some optimization criteria, and the compensator $k_f^*(s)$ is designed by the transfer function $G_p(s)$ of nominal operation case, within uncertain or non-linear model P, so that control system outputs under other operation cases are the same as nominal operation case.

PIDC control systems is shown in Figure 6.

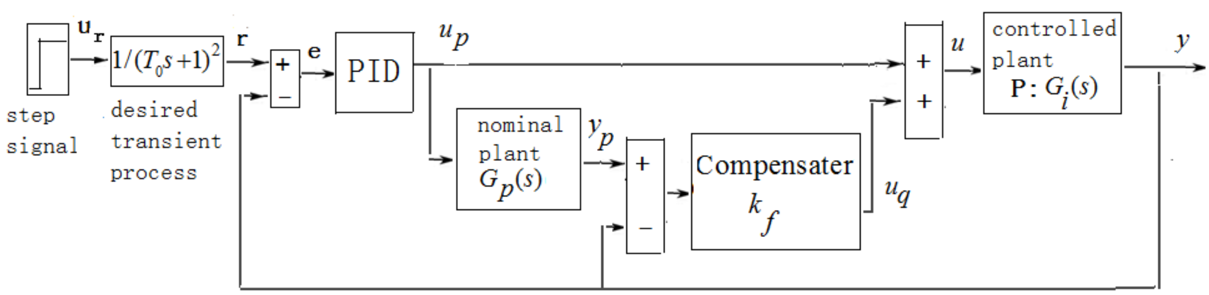

Figure 6 PIDC control systems with compensator

In Figure 5, $PID(s) = k_r(k_p + k_i/s + k_d s/(t_d s+1))$, the controlled plant P is an uncertain or non-linear plant (time–variant models), $r = U_r/(T_0 s+1)^2$ is designed to adjust amplitude of control function u and transient process time, U_r is step signal with the amplitude $|U_r| = r$.

Definition 1: The controlled plant P is an uncertain or non-linear plant, which consist of many operation cases based on the transform function $G_i(s)$ of the uncertain plant P, $i = 0,1,...n,$ or $G_i(s)$ are the transform functions under some operation cases after linearizing the non-linear plant P.

Definition 2: Let $G_i(s) \in P$, $i = 0,1,...n,$ and

$$G_i(s) = b_i e^{(-s\tau_i)} / (a_{i2}s^2 + a_{i1}s + 1) \tag{3}$$

is stable.

Definition 3: Let $G_p(s) \in P$, $G_p(s)$ is the nominal operation case of P, the parameter $K^* = [k_r^*, k_p^*, k_i^*, k_d^*, t_d^*]$ of $PID*$ is designed based on $G_p(s)$, $u_p = PID * e(t)$, where $e(t) = r(t) - y(t)$, the output y_p of closed loop system based on $G_p(s)$, $y_p = G_p(s)u_p$ meets the control system quality index according to some optimization criterions, and the output y_p is stable.

Theorem 1: $PID*$ controller based on $G_p(s)$ is designed under some optimization criterions in Figure 5, the output u_p of $PID*$ makes that the closed loop system output $y_p = G_p(s)u_p$ meets the control system quality index, where $u_p = PID*e,\ e = r - y$. a compensator k_f is designed, so that

$$y_i(t) = y_p(t),\ i = 1, 2, ...n,$$

then the compensator k_f

$k_f = G_i^{-1}(s)$, where $G_i(s) \in P,\ \ i = 1, 2, ...n$

Proof: From Definition. 3 and Figure 5,

$y_p = G_p(s)u_p$, because of $y_i = G_i(s)u_p \neq y_p$,

so the compensator k_f is added, The whole control output u

$$u = u_p + u_q,$$

so that every closed loop output y_i of control system in all operation cases based on $G_i(s)$, $\forall i,\ i = 1, 2...n,\ G_i(s) \in P$, equal output y_p of the nominal case $G_p(s)$, i.e. $y_i = y_p$,

From Figure 5,

$$\begin{aligned}
&(u_p + u_q)G_i(s) = y_p \\
&(u_p + (y_p - y_i)k_f)G_i(s) = y_p \\
&G_i(s)u_p + (y_p - y_i)k_f G_i(s) = y_p \\
&(y_p - y_i)k_f G_i(s) = y_p - G_i(s)u_p \\
&(y_p - y_i)k_f G_i(s) = y_p - y_i
\end{aligned} \tag{4}$$

so,

$$k_f(s) = G_i^{-1}(s),\quad \forall G_i(s) \in P, i = 1, 2...n,, \tag{5}$$

The theorem has been proven.

Theorem 2:

If the compensator $k_f^*(s)$ is designed as a strictly proper transfer function based on $k_f(s) = G_p^{-1}(s)$, $k_f^*(s) = thr(s)k_f(s),$ where $thr(s) = 1/(t_m s^2 + 2t_m s + 1)$ and, t_m is a small and suitable time contestant, then the compensator $k_f^*(s) = thr(s)k_f(s)$ can make all closed outputs y_i based on $G_i(s) \in P$, $i = 1, 2...n,$ are stable and $\lim_{t\to\infty} |(y_i(t) - y_p(t)| = 0, i = 1, 2, ...n.$

Proof: From Figure 5 and Eq. (2), let $k_f = G_p^{-1}(s)$

$$\begin{aligned}
&(u_p + (y_p - y_i)k_f)G_i(s) = y_p \\
&y_i + (y_p - y_i)G_p^{-1}(s)G_i(s) = y_p \\
&y_i(1 - G_p^{-1}(s)G_i(s)) = y_p(1 - G_p^{-1}(s)G_i(s)) \\
&y_i = y_p,\ i = 0, 1, 2, ...n
\end{aligned} \tag{6}$$

From Figure 5, $y_i = k_f^*(s)G_i(s)y_p / (1 + k_f^*(s)G_i(s)),$ $i = 1, 2...n$ the closed loop system output, where $thr(s) = 1/(t_m s^2 + 2t_m s + 1)$, t_m is a small and suitable time contestant, $k_f^*(s) = thr(s)k_f(s)$ is a strictly proper transfer function based on $k_f(s) = G_p^{-1}(s)$, let sensitivity function $S = 1 - G_i(s)Q,\quad Q = k_f^*(s)/(1 + k_f^*(s)G_i(s))$, then $k_f^*(s) = (1 - QG)^{-1}Q$ meets Youla parametrisation Formulas [7], so $k_f^*(s)$ can make all closed outputs y_i based on $G_i(s) \in P$, $i = 1, 2...n,$ are stable and $\lim_{t\to\infty} |(y_i(t) - y_p(t)| = 0, i = 1, 2, ...n.$

The theorem has been proven.

The compensator k_f in theory:

$$k_f = (a_{p2}s^2 + a_{p1}s + 1)e^{s\tau_p} / (b_p) \tag{7}$$

because $\tau_q = \max(\tau)$ is maximum dead time in P, $G_q(s) = b_q e^{-s\tau_q} / (a_{q2}s^2 + a_{q1}s + 1) \in P,$ optimization of $PID*$ under nominal operation case $G_p(s) = b_p e^{-s\tau_p} / (a_{p2}s^2 + a_{p1}s + 1)$, we have designed the parameters of $PID*$ for τ_p, so substituting $f(\Delta\tau) = f(\tau_q - \tau_p)$ for τ_p in Eq. (7).

When using Eq. (5), $e^{s\tau_p} \cong e^{sf(\Delta\tau)}$ need to be converted to power series. Eq. (5) must be designed as a strictly proper transfer function. A simple algorithm is designing transfer function $trs(s) = 1/(t_m{}^2 s^2 + 2t_m s + 1),$ where t_m is a small and suitable time contestant, compensator $k_f^*(s),$

$$k_f^*(s) = thr(s)k_f \tag{8}$$

$$e^{s\tau_p} \cong e^{sf(\Delta\tau)} = e^{(s\Delta\tau)} = 1 + s\Delta\tau + 0.5(s\Delta\tau)^2$$

so, the compensator $k_f^*(s)$,

$$\begin{aligned}
&k_f^*(s) = thr(s)(a_{p2}s^2 + a_{p1}s + 1)e^{s\tau_p} / (b_p) \\
&= ((a_{p2}s^2 + a_{p1}s + 1)(1 + s\Delta\tau + 0.5(s\Delta\tau)^2)) / \\
&[(bp(t_m s^2 + 2t_m s + 1)(1 + s\varsigma\Delta\tau + 0.5s^2(\varsigma\Delta\tau)^2]
\end{aligned} \tag{9}$$

where ς is small and suitable constant number.

3 Uncertain System with Model-free Design Based on Newton's Laws of Motion [1, 2, 4, 8, 14, 15]

Design of control systems needs a mathematics model of controlled plant in general, however the model is not exactly obtained and the model is uncertain, these limits led to the poor control system performance.

When a motion (/reaction) speed of controlled plant process (/body) is greatly slower than speed of light, its motion process can described by Newton's Laws of Motion.

Newton's Laws of Motion are described as follows:

The first law: In an inertial frame of reference, an object either remains at rest or continues to move at a constant speed, unless acted upon by a force.

The second law: In an inertial reference frame, the vector sum of the forces F on an object is equal to the mass m of that object multiplied by the acceleration of the object:

$F = ma$ (It is assumed here that the mass m is constant).

The third law: When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body.

3.1 OUAM Filter (Based on Uniform Acceleration Movement)

The second order controlled plant process is a typical controlled plant process in control theory and engineering, it is represented as:

$$\ddot{y} = f(y, \dot{y}, w(t)) + bu \tag{10}$$

where, $w(t)$ is an unknown random disturbance.

Its state equation is represented as:

$$\dot{y}_1 = y_2 \tag{11}$$

$$\dot{y}_2 = f(y_1, y_2, w(t)) + bu \tag{12}$$

$$y = y_1 \tag{13}$$

The uniformly acceleration motion in Newton's Laws of Motion is:

$$S = S_0 + V_0 t + 0.5at^2 \tag{14}$$

$$\dot{S} = V = V_0 + at \tag{15}$$

$$\ddot{S} = \dot{V} = a \tag{16}$$

where S, V, a are respectively the position, speed and acceleration of the body motion (process/reactions).

For a controlled plant process, assuming $z(1)$, $z(2), \cdots, z(k)$, the k measurement output data are obtained from the controlled plant output $y(k)$, sample (/control) period is t_s, the measurement equation is

$$z(k) = y(k) + v(k) \tag{17}$$

where:

$E[v^2(k)] = r_2$

$E[v(k)] = 0$

$v(k)$ is white noise, we can estimate the controlled plant output $y(k)$ using Eq. (17) from $z(1)$, $z(2), \cdots, z(k)$ when t_s is very short,

$$\hat{y}(k) = \hat{y}(k-1) + t_s \dot{\hat{y}}(k-1) + 0.5 t_s^2 \ddot{\hat{y}}(k-1) \tag{18}$$

$$\dot{\hat{y}}(k) = \dot{\hat{y}}(k-1) + t_s \ddot{\hat{y}}(k-1) \tag{19}$$

$$\ddot{\hat{y}}(k) = \ddot{\hat{y}}(k-1) \tag{20}$$

where $\hat{y}(k)$, $\dot{\hat{y}}(k)$ and $\ddot{\hat{y}}(k)$ are respectively the estimated values of $y(k)$, $\dot{y}(k)$ and $\ddot{y}(k)$ at time k.

Let

$$\hat{Y}(k) = \begin{bmatrix} \hat{y}(k) \\ \dot{\hat{y}}(k) \\ \ddot{\hat{y}}(k) \end{bmatrix}$$

and

$$\phi = \begin{bmatrix} 1 & t_s & t_s^2/2 \\ 0 & 1 & t_s \\ 0 & 0 & 1 \end{bmatrix} \tag{21}$$

Eq. (5) can be written as:

$$\hat{Y}(k) = \phi \hat{Y}(k-1) \tag{22}$$

To improve estimate accuracy, the compensating for random disturbance is into to acceleration estimate $\ddot{\hat{y}}(k)$:

$$\ddot{y}(k) = \ddot{y}(k-1) + w(k-1) \tag{23}$$

where:

$E[w(k)] = 0$, $E[w^2(k)] = r_1$

Eq. (23) becomes:

$$\hat{Y}(k) = \phi\hat{Y}(k-1) + \Gamma w(k-1) \tag{24}$$

Where:

$\Gamma = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$

The measurement equation (4) is written as:

$$z(k) = c\hat{Y}(k) + v(k), \tag{25}$$

where:

$c = [1 \quad 0 \quad 0]$

$E[w(k)] = 0, E[v(k)] = 0$

$E[y(0)] = m$, $E[w(k)v^T(j)] = 0$

$E[_v{}^2(k)] = r_2, E[w^2(k)] = r_1$,

$E[y(0)] = m$

$V_{arx}[y(0)] = P_0$

Kalman filter theory can be used for state equation (24) and measurement equation (25), ϕ and Γ are constant matrixes,

We calculate,

$$(\phi, c) = [c^T \mid \phi^T c^T \mid (\phi^T)^2 c^T] = \begin{bmatrix} 1 & 1 & 1 \\ 0 & t_s & 2t_s \\ 0 & t_s^2/2 & 2t_s^2 \end{bmatrix}$$

$$(\phi, \Gamma) = [\Gamma \mid \phi\Gamma \mid \phi^2\Gamma] = \begin{bmatrix} 0 & t_s^2/2 & 2t_s^2 \\ 0 & t_s & 2t_s \\ 1 & 1 & 1 \end{bmatrix}$$

$rank(\phi, \Gamma) = 3$, $rank(\phi, c) = 3$; so discrete systems (24) and (25) are completely controllable and observable, when t_s is very short and filtering time is very long, the covariance matrix,

$$\lim_{k\to\infty} P(k) = P$$

Gain matrix

$$\lim_{k\to\infty} K(k) = K^*,$$

$$K(k) \to K^* = [\alpha, \beta, \gamma]^T.$$

We have the following time discrete filter to an unknown controlled plant according to Kalman filter theory.

$$\begin{cases} \hat{y}(k) = \hat{y}(k-1) + t_s\dot{\hat{y}}(k-1) + 0.5t_s^2\ddot{\hat{y}}(k-1) + \alpha t_s(z(k) - \hat{y}(k-1)) \\ \dot{\hat{y}}(k) = \dot{\hat{y}}(k-1) + t_s\ddot{\hat{y}}(k-1) + \beta t_s(z(k) - \hat{y}(k-1)) \\ \ddot{\hat{y}}(k) = \ddot{\hat{y}}(k-1) + \gamma t_s(z(k) - \hat{y}(k-1)) \end{cases} \tag{26}$$

Because Kalman filter is an unbiased estimation with $\min J = \hat{E}[\tilde{x}^T(k \mid j)\tilde{x}(k \mid j) \mid Z_j]$, where $\tilde{x}(k \mid j) = x(k) - \hat{x}(k \mid j)$ is the mean Square Error of $x(k)$, $Z_j = \{z_1, z_2, \ldots z_n\}$.

$P(k \mid j) = \hat{E}[\tilde{x}^T(k \mid j)\tilde{x}(k \mid j) \mid Z_j]$, so the filter Eq. (26) based on Kalman filter also is unbiased estimate.

Let $\hat{y}_1(k) = \hat{y}(k)$, $\hat{y}_2(k) = \dot{\hat{y}}(k)$, $\hat{y}_3(k) = \ddot{\hat{y}}(k)$ in Eq. (26), and Eq. (26) is operated as follows:

$$\begin{aligned} &(\hat{y}_1(k) - \hat{y}_1(k-1))/t_s = \hat{y}_2(k-1) \\ &+0.5t_s\hat{y}_3(k-1) + \alpha(z(k) - \hat{y}(k-1)) \\ &(\hat{y}_2(k) - \hat{y}_2(k-1))/t_s = \hat{y}_3(k-1) \\ &+\beta(z(k) - \hat{y}(k-1)) \\ &(\hat{y}_3(k) - \hat{y}_3(k-1))/t_s = \gamma(z(k) - \hat{y}(k-1)) \end{aligned} \tag{27}$$

Let $t_s \to 0$ in Eq. (27), we have the time continuous filter

$$\begin{aligned} \dot{\hat{y}}_1 &= \hat{y}_2 + \alpha(z - \hat{y}) \\ \dot{\hat{y}}_2 &= \hat{y}_3 + \beta(z - \hat{y}) \\ \dot{\hat{y}}_3 &= \gamma(z - \hat{y}) \end{aligned} \tag{28}$$

where,

$$A = \begin{bmatrix} -\alpha & 1 & 0 \\ -\beta & 0 & 1 \\ -\gamma & 0 & 0 \end{bmatrix} \tag{29}$$

Relating Eq. (2), control variable bu is added to Eq. (28), Eq. (28) became

$$\begin{cases} \dot{\hat{y}}_1 = \hat{y}_2 + \alpha(z - \hat{y}) \\ \dot{\hat{y}}_2 = \hat{y}_3 + \beta(z - \hat{y}) + bu \\ \dot{\hat{y}}_3 = \gamma(z - \hat{y}) \end{cases} \tag{30}$$

Eq. (30) is written as:

$$\dot{\hat{Y}} = A\hat{Y} + Bu_c \\ \hat{y} = C\hat{Y} \tag{31}$$

Where

$$\hat{Y} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \end{bmatrix}$$

$$C = [1 \quad 0 \quad 0] \tag{32}$$

$$B = \begin{bmatrix} \alpha & 0 \\ \beta & b \\ \gamma & 0 \end{bmatrix}, \quad u_c = \begin{bmatrix} z \\ u \end{bmatrix} \tag{33}$$

A is represented in Eq. (29)

$$\hat{y}_1 = \hat{y}; \quad \hat{y}_2 = \dot{\hat{y}}; \quad \hat{y}_3 = \ddot{\hat{y}}$$

t_s is sampling time, and u is the control input.

Eq. (30) is the time continuous observer of an unknown disturbance controlled plant with control input u for the control system represented in Eq. (2), Eq. (30) is simply called OUAM observer (based on Uniform Acceleration Movement).

Simultaneously, We have the following observer system for a controlled plant process with an acceleration $a = 0$,

$$\dot{\hat{y}}_1 = \hat{y}_2 + \alpha(z - \hat{y}) + bu \\ \dot{\hat{y}}_2 = \beta(z - \hat{y}) \\ \hat{y} = \hat{y}_1 \tag{34}$$

We have the following observer system for a controlled plant process with varying acceleration a.

$$\begin{bmatrix} \dot{\hat{y}}_1 \\ \dot{\hat{y}}_2 \\ \dot{\hat{y}}_3 \\ \dot{\hat{y}}_4 \end{bmatrix} = \begin{bmatrix} -\alpha & 1 & 0 & 0 \\ -\beta & 0 & 1 & 0 \\ -\gamma & 0 & 0 & 1 \\ -\theta & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \\ \hat{y}_4 \end{bmatrix} + \begin{bmatrix} \alpha & 0 \\ \beta & 0 \\ \gamma & \lambda \\ \theta & 0 \end{bmatrix} \begin{bmatrix} z \\ u \end{bmatrix} \tag{35}$$

$$\hat{y} = \hat{y}_1$$

In Eq. (30), Eq. (34) and Eq. (35), the disturbances on systems are assumed as white noise. In generally, the disturbances can be classified as white noise or as colored noise, the model of system with colored noise disturbances is represented as:

$$x(k+1) = \phi(k+1,k)x(k) + \Gamma(k)\vec{w}(k), (\mathrm{k} \ge 0) \\ z(k) = c(k)x(k) + \vec{v}(k), (\mathrm{k} \ge 0) \tag{36}$$

where, $\vec{w}(k)$ and $\vec{v}(k)$ are the colored noise series, $\vec{w}(k)$ and $\vec{v}(k)$ can be produced by a white noise series.

$$\vec{w}(k+1) = \phi_1(k+1,k)\vec{w}(k) + \xi_1(k), \\ \vec{v}(k+1) = \phi_2(k+1,k)\vec{v}(k) + \xi_2(k) \tag{37}$$

where $\{\vec{w}(k)\}$ and $\{\vec{v}(k)\}$ are one order Markov series, $\xi_1(k)$ and $\xi_2(k)$ are respectively the white noise series with zero mean.

Their covariance: $E[\xi_1(k)\xi_1^{\mathrm{T}}(j)] = s_1(k)\delta_{k,j}$, $E[\xi_2(k)\xi_2^{\mathrm{T}}(j)] = s_2(k)\delta_{k,j}$. New state variables are applied with $\vec{w}(k)$ and $\vec{v}(k)$ under the colored noise series:

$$x^*(k+1) = \begin{bmatrix} x(k+1) \\ \vec{w}(k+1) \\ \vec{v}(k+1) \end{bmatrix},$$

the new model with equations of system and measure.

$$x^*(k+1) = \begin{bmatrix} x(k+1) \\ \vec{w}(k+1) \\ \vec{v}(k+1) \end{bmatrix}$$

$$x^*(k+1) = \phi^*(k+1,k)x^*(k) + \xi(k) \\ z(k) = c^*(k)x^*(k) \\ \xi(k) = \begin{bmatrix} 0 \\ \xi_1(k) \\ \xi_2(k) \end{bmatrix}, \ c^*(k) = [c(k) \ 0 \ 1], \tag{38}$$

$$\phi^*(k+1,k) = \begin{bmatrix} \phi(k+1,k) & \Gamma(k) & 0 \\ 0 & \phi(k+1,k) & 0 \\ 0 & 0 & \phi(k+1,k) \end{bmatrix}$$

3.2 Uncertain System With Model-free Design Based on Newton's Laws of Motion

Newton's Laws of Motion are consist of 3 variables, the position, speed and acceleration of the body motion (process/reactions). When the 3 variables can be estimated and calculated, the control system design and operation are became easy and convenient. The control system quality will be improved with help of Newton's Laws of Motion. The designed Control System based on Newton's Laws of Motion is as follows.

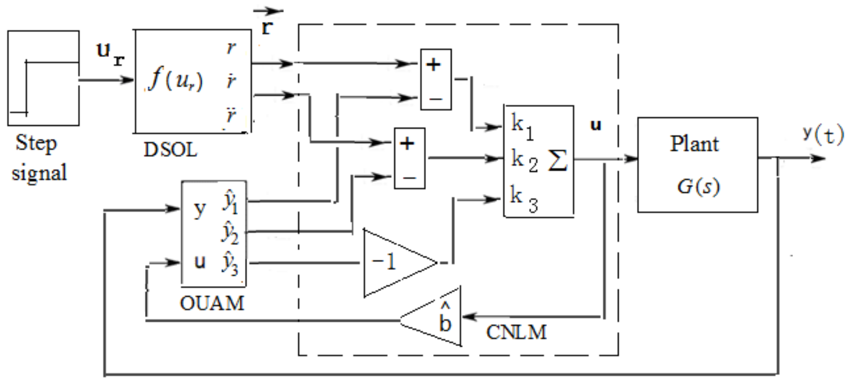

Figure 7 CSMFDNLM (Control System with Model-Free design based on Newton's Laws of Motion)

The designed Control Systems are consisted of these modules: DSOL (Desired System Output Locus), OUAM (observer based on Uniform Acceleration Movement) and CNLM (controller based on Newton's Laws of Motion).

3.2.1 Design Desired System Output Locus (DSOL)

In module DSOL of Figure 6, we have the transfer function between desired system output locus r and the input $u_r(s)$,

$$r(s)/u_r(s) = 1/(cs^2 + ds + 1) \tag{39}$$

in Eq. (39) by setting suitable parameters c, d, the desired system output locus r is obtained, *where* $u_r(s)$ is a step signal with amplitude $|u_r(s)| = r,\ u_r(s) = ru_0$, u_0 is an unit step signal.

The control system output y monotone smooth tracks control input r, The time T when y arrives at 98% r, w_n is defined as the desired time T of the transition process of control system output y.

Eq. (40) in state equation is represented as:

$$\begin{aligned} \dot{r}_1 &= r_2 \\ \dot{r}_2 &= -dr_2/c - r_1/c + u_r/c \\ &= f(r_1, r_2) + ru_0/c \\ r &= r_1 \end{aligned} \tag{40}$$

where,

$$c = 1/w_n^2,\ d = 2\xi/w_n,\ \xi=1 \tag{41}$$

r is amplitude of desired system output y, w_n is the natural frequency of control system, which represents system operation speed, and $\xi = 1$ is damping coefficient of the system.

The desired time T of the transition process of control system output y is related to w_n,

$$T = 7/w_n \tag{42}$$

In General, $T = nt_s$ $n \in (800\ 1200)$, T implies that control system output y arrives its desired value after running n times at control period t_s.

Designing the parameters of Eq. (39), (40), (41) and (42), so that The control system output y monotone smooth tracks control input r, y arrives at 98% $|r|$ in the time T.

3.2.2 Design OUAM State Observer

The state observer is designed by Eq. ((31)).

Its characteristic equation by Eq. (31)

$$m_A(s) = |sI - A| = s^3 + \alpha s^2 + \beta s + \gamma$$

If $\alpha > 0$, $\beta > 0$, $\gamma > 0$ and $\alpha\beta > \gamma$, then time continuous observer system (31) is stable, according to Routh and Hurwitz's stability criterion.

Let $m_A(s) = (s + s_1)^3$ =0, its 3 solutions at $-s_1$ and $s_1 > 0$ then

$$\alpha = 3s_1,\ \beta = 3s_1^2,\ \gamma = s_1^3 \tag{43}$$

Suitable select s_1, so that $s_1 = f_{11}(a_{11}, 1/T)$, then

$$\alpha = 3s_1 = f_{11}(a_{11}, 1/T) = a_{11}/T; \tag{44}$$

$$\beta = 3s_1^2 = f_{12}(a_{12}, 1/T^2) = a_{12}/T^2 \tag{45}$$

$$\gamma = s_1^3 = f_{13}(a_{13}, 1/T^3) = a_{13}/T^3 \tag{46}$$

3.2.3 Design Controller (CNLM) [2-6]

From Eq. (10) and Newton's Laws of Motion, we have the acceleration,

$$a = \ddot{y} = f(y, \dot{y}, w(t)) + bu \tag{47}$$

From Eq. (47), the control force u is obtained,

$$u = (a(s) - f(y, \dot{y}, w(t)))/b \tag{48}$$

Take u in Eq. (47) to Eq. (11), Eq. (12) and Eq. (13), we have:

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= a \\ y &= y_1 \end{aligned} \tag{49}$$

The transfer function between system output y and acceleration a for Eq. (49).

$$y(s)/a(s) = g_{ya}(s) = C^T[SI - A]^{-1}B = 1/s^2 \tag{50}$$

where,

$$C^T = [1\ \ 0],\ A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The transfer function $g_{ya}(s)$ is a double integration function, so that poles position of the system can be allocated by acceleration a. Under assuming $y_3 = f(y_1, y_2, w(t))$, The system Eq. (2) is converted to

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= y_3 + bu \\ \dot{y}_3 &= f(y_1, y_2, w(t)) \\ y &= y_1 \end{aligned} \tag{51}$$

Because the system Eq ((30)) is an unbiased estimate system, comparing Eq. (51) with Eq. (30), we have:

$\hat{y}_1 \to y_1 = y$; $\hat{y}_2 \to y_2 = \dot{y}$; $\hat{y}_3 \to y_3 = f(y_1, y_2, w(t))$.

so the $\hat{y}_3$ in Eq. (30) is an unbiased estimation to the uncertain function $f(y_1, y_2, w(t))$ in the system Eq. (10).

Design the acceleration $a(s)$ in Eq. (50).

$$a(s) = K_1(r - y) + K_2(\dot{r} - \dot{y})\,. \tag{52}$$

Eq. (52) applies errors of $(r - y)$ and $(\dot{r} - \dot{y})$ as the drive factors of the acceleration a(s) to improve control system performance, where y and r are respective control system output and desired system output.

Take $a(s)$ in Eq. (52) to Eq. (50), because of unbiased estimate system Eq. (13) then we have:

$$\begin{aligned} \hat{y} &= (K_1(r - \hat{y}_1) + K_2(\dot{r} - \hat{y}_2))/s^2 \\ &= (K_1(r - \hat{y}) + K_2(rs - \hat{y}s))/s^2\,. \end{aligned}$$

The transfer function $H(s)$ between control system output $\hat{y}$ and control command r

$$\hat{y}/r = H(s) = (K_1 + K_2 s)/(s^2 + K_2 s + K_1)$$

The closed loop system characteristic equation.

$$m_H(s) = (s^2 + K_2 s + K_1)$$

Design poles position at $-s_2$, and s_2>0, for the closed loop system characteristic equation.

$$m_H(s) = (s + s_2)^2 = s^2 + 2s_2 s + s_2^2$$

so,

$$K_1 = s_2^2, K_2 = 2s_2,$$

Suitable select s_2 so that,

$$K_1 = s_2^2 = f_{22}(a_{22}, 1/T^2) = a_{22}/T^2 \tag{53}$$

$$K_2 = 2s_2 = f_{21}(a_{21}, 1/T) = a_{21}/T \tag{54}$$

to meet the performance target of the control system.

From Eq. (48), Eq. (53) and Eq. (54), the control force function:

$$u = (K_1(r - \hat{y}_1) + K_2(\dot{r} - \hat{y}_2) - f(\hat{y}_1, \hat{y}_2, w(t)))/\hat{b} \tag{55}$$

where, $\hat{b}$ is an estimation to b in Eq. (10) which is static gain of the controlled plant (process), b can be estimated by open loop output of the controlled plant under step signal input.

Because of

$$\hat{y}_3 \to f(y_1, y_2, w(t))$$

so

$$u \approx (k_1(r - \hat{y}_1) + k_2(\dot{r} - \hat{y}_2) - k_3\hat{y}_3. \tag{56}$$

where

$$k_1 = K_1/\hat{b}, \ ,k_2 = K_2/\hat{b}, \ k_3 = 1/\hat{b} \tag{57}$$

$$k = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}^T,$$

All designed parameters are only related to the desired transient process time T of system output without controlled plan model in the paper.

Eq. (56) is the designed controller in Figure 6 (CSMFDNLM), as $\hat{y}_3 \to f(y_1, y_2, w(t))$, so u includes the factor $-f(y_1, y_2, w(t))$ and eliminates uncertain and disturbance factors of the system. The control system (CSMFDNLM) improves control qualities and increases robust performance of system.

Calculate procedure of parameters in CSMFDNLM

After the desired time T of transition process of the control system is determined by control engineer,

Start calculation as follows

Steps:

(1). Calculate w_n, c, d by Eq. (41) and Eq. (42) in DSOL;

(2) Calculate K_1, K_2 by Eq. (53) and Eq. (54);

Calculate the controller parameters $k = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}^T$ by Eq. (57), $\hat{b}$ can be calculated, after estimation to b using open loop output of the controlled plant under step signal input.

(3). Calculate observer parameters α, β, γ. by Eq. (44), Eq. (45) and Eq. (46).

If the T can't be determined by control engineer, then the process time $T = nt_S$ is set, where t_S is the control period (or sample time period of DCS.), and $n \in (800 - 1500)$. Control engineer may try to increase n from n = 800 to 1200, then repeat above calculation steps using Computer Aided Design until Satisfaction control performance is obtained.

Comparing between the shapes of DSOL curve and control system output curve, if there are bigger difference in process time T, then the time T needs to been adjusted until the 2 curves sufficient closed.

Finally, we can use Gain Margin and phase margin with nyquist plots to analysis and synthesis the quality of *CSMFDNLM.*

4 Classical Mechanics Unity Principle of Control System Design [3, 5, 10, 12, 13, 15]

Now, although there are many methods of designing control systems, for examples, PID and PIDC system, state feedback control, pole allocation, adaptive control, GPC, MPC control, various optimization control etc. These control systems operation speed are great slower than light speed, so that Newton's Laws of Motion can been applied to the design and analysis of the control systems. The control systems are of Classical mechanics unity.

A second order system is a typical controlled plant with 2 state variables, 3 parameters of PID controller need to be optimized, the second system is expanded to the third order system.

Let the transfer function of the second controlled plant,

$$y/u = G(s) = k/(as^2 + bs + 1) \tag{58}$$

Its differential equation is:

$$\ddot{y} = -y/a - b\dot{y}/a + ku/a,$$

second differentiation on $\ddot{y}$, we have:

$$\dddot{y} = -\dot{y} - b\ddot{y}/a + k\dot{u}/a \tag{59}$$

let $e = e_1 = r - y$,

$$\begin{aligned} e_2 &= \dot{r} - \dot{y}, \\ e_3 &= \ddot{r} - \ddot{y}, \end{aligned} \tag{60}$$

where, r is the set value.

Let $v_r = \dot{u}$, above state equation becomes:

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1/a & -b/a \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ k/a \end{bmatrix} v_r \tag{61}$$

For above the third order system, state feedback control, pole allocation, adaptive control, GPC, MPC control, quadratic binary optimization and various optimization control methods etc. can been used to obtain the solutions of controller parameters k_1, k_2, k_3 .

$$v_r = k_1 e_1 + k_2 e_2 + k_3 e_3 \tag{62}$$

considering: $v_r = \dot{u}$; so,

$$u = \int v_r dt = \int k_1 e dt + k_2 e + k_3 de/dt \,, \tag{63}$$

Eq. (63) is written in PID controller:

$$G_{PID}(s) = k_1/s + k_2 + k_3 s \tag{64}$$

revising above Equation as a real PID controller:

$$\begin{aligned} G_{PID}(s) &= k_1/s + k_2 + k_3 s/(nt_s s+1) \\ &= k_p + k_i/s + k_d s/(t_d s+1) \end{aligned} \tag{65}$$

where, $n \in [1\ 5)$, $t_d = nt_s$

Assume parameters $k = [k_1 \quad k_2 \quad k_3]^T$ in CNLM of CSMFDNLM.

So, the parameters $[k_p\ k_i\ k_d]$ of PID controller correspond with $k = [k_1 \quad k_2 \quad k_3]$ in CNLM of CSMFDNLM.

$$k_p = k_2,\ k_i = k_1,\ k_d = k_3; \tag{66}$$

Eq. (66) reveals the classical Newton's Laws of motion unity principle among state feedback control of modern control theory and classical PID control theory.

5 Simulation and Engineering Application Examples

5.1 PID Control System with Improved Smith Predictor

The control system is shown in Figure 3, the controlled plant G(s)=0.7e^{-30s} /(50s+1)=G$_0$(s) e^{-30s}, control (sample) period t_s=0.1(s), $r = U_r/(T_0 s+1)^2$ is designed to adjust amplitude of control function u and transient process time, U_r is unit step signal with the amplitude $|U_r|$=1, the PID controller is designed based on G$_0$(s)= 0.7/(50s+1), PID(s)=0.37(2.56+0.05/s), where $k_r = 0.37$, based on k_r=f(τ/T$_t$), so that output y_0 (in Figure 4) is shifted time τ to y based on G(s) (in Figure 5). From the Figure 1, $k_r = 0.9$, k_r=0.23 are not suitable to some optimization. The improved Smith predictor with the function of k_r and y_0 (in Figure 4) shifted time τ to y (in Figure 5) so that design of the controlled plant with the loop of pure delay is more simpler.

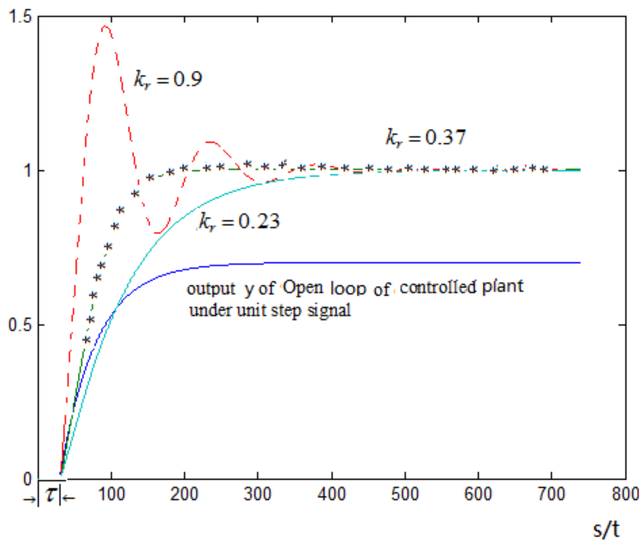

Figure 8 Function of improve Smith predictor with k_r

5.2 Application of PIDC Control System with Compensator

A thermal engineering control system in a electricity power factory is designed for an uncertain controlled plant process P in Figure 5, the transfer functions of 3 operation cases are

$$\begin{aligned} G_1(s) &= 0.5e^{-24s}/(553.6s^2 + 47.1s+1)\,, \\ G_2(s) &= 1.5e^{-46s}/(1175.5s^2 + 68.6s+1)\ ; \\ G_3(s) &= 0.75e^{-32s}/(816.3s^2 + 57.1s+1)\,; \end{aligned}$$

selecting $G_p(s) = G_3(s)$, $G_p(s), G_1(s)$ and $G_2(s) \in P$, control period (sample time) $t_s = 0.2(s)$, T$_0$=7(s) , U$_r$ is the step signal with amplitude 1(after processing normalization). The output y of control system needs to reach 1 in 210(s).

$G_p(s)$ is the transfer function of nominal operation case within model P.

The parameters $K^* = [k_r^*, k_p^*, k_i^*, k_d^*, t_d^*]$ of $PID*$ controller.

$K^* = [k_r^*, k_p^*, k_i^*, k_d^*, t_d^*] = [0.3, 1.75, 0.035, 16, 7]$ is designed based on $G_p(s)$

From Exp. (5), and Exp. (6), we have the compensator $k_f^*(s)$.

$$k_f^*(s) = [(816.33s^2 + 57.14s + 1.0)/(0.24s^2 + 1.20s + 1.50)][(0.041s^2 + 0.29s + 1.0)/(9s^2 + 6s + 1)]$$

The control system output y under unit step signal input U_r are shown in Figure 9, Figure 10 and Figure 11 respectively with $G_p(s), G_1(s)$ and $G_2(s)$ using the same parameters $K^* = [k_r^*, k_p^*, k_i^*, k_d^*, t_d^*]$ of $PID*$. $k_f^*(s) = 0$ (without the compensator) in Figure 3. (i.e, PID control system with improved Smith predictor without the compensator).

The compensator, in PIDC control systems (Figure 5),

$$k_f^*(s) = [(816.33s^2 + 57.14s + 1.0)/(0.24s^2 + 1.20s + 1.50)]*[(0.041s^2 + 0.29s + 1.0)/(9s^2 + 6s + 1)]$$

Figures 9, 10 and 11 compared the operation results of between PID control system with improved smith predictor (Figure 2) and PIDC control system with compensator (Figure 5).

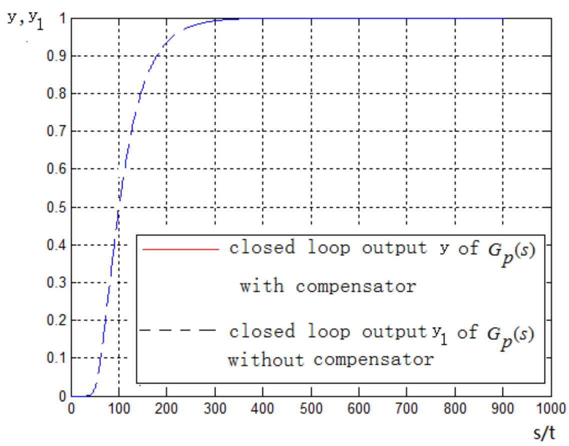

Figure 9 Output of closed loop system of $G_p(s)$ with /without compensator (curve y is overlaid by curve y_1)

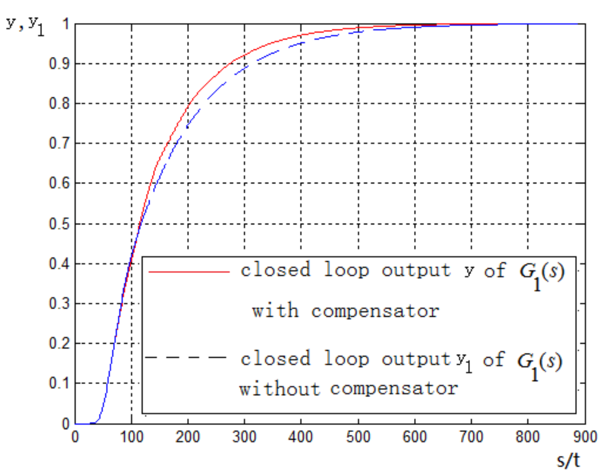

Figure 10 Output of closed loop system of $G_1(s)$ with/without compensator

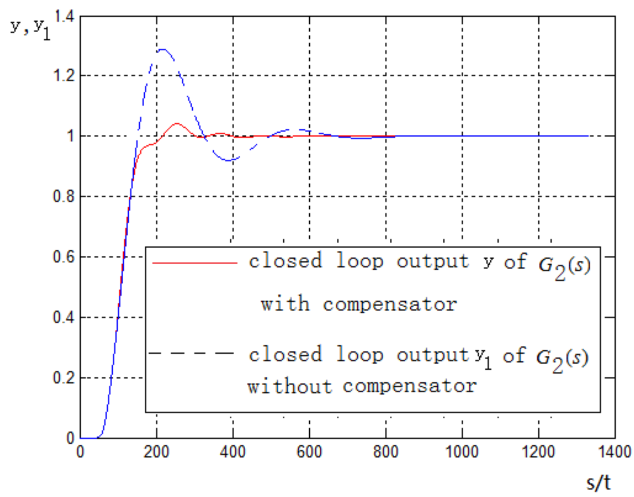

Figure 11 output of closed loop system of $G_2(s)$ with/without compensator

5.3 Application of CSMFDNLM

A control system in Figure 5, is designed for an uncertain controlled plant process P:, the transfer functions of 3 operation cases are

$$G_1(s) = 0.5e^{(-15s)} / (553.6s^2 + 47.1s + 1)$$
$$G_2(s) = 0.75e^{(-25s)} / (816.3s^2 + 57.1s + 1)$$
$$G_3(s) = 1.5e^{(-35s)} / (1175.5s^2 + 68.6s + 1)$$

$G_2(s)$ is selected as the model of nominal case, control period (sample time) $t_s = 0.2(s)$. The desired time of the transition process of control system output, $T = 1250t_s = 250(s)$, the natural frequency of control system, $w_n = 7/T = 0.028$, which represents system operation speed with $\xi = 1$, damping coefficient of the system. Calculating parameters of OUAM Observer,

$\alpha = 0.875, \beta = 0.255, \gamma = 0.0248$, parameters of CNLM controller $k_1 = 1.44, k_2 = 82.286, k_3 = 1176$. $k_r = 0.14$.

Parameters of PID control system with improved smith predictor (Figure 3): $T_0 = 8, k_r = 0.36$, $PID(s) = 1.42 + 0.029/s + 11s/(3s+1)$.

Comparing the operation results of between PID control system with improved smith predictor (Figure 3) and CSMFDNLM (Figure 7) in 3 operation cases are shown in Figures 12, 13 and 14.

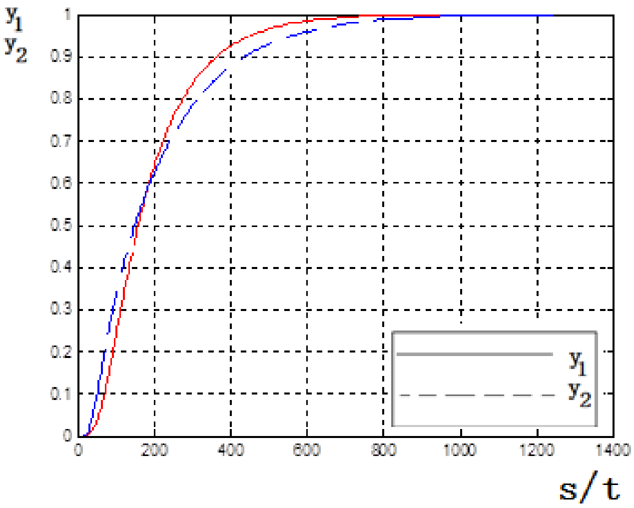

Figure 12 Operation results of PID control system and CSMFDNLM

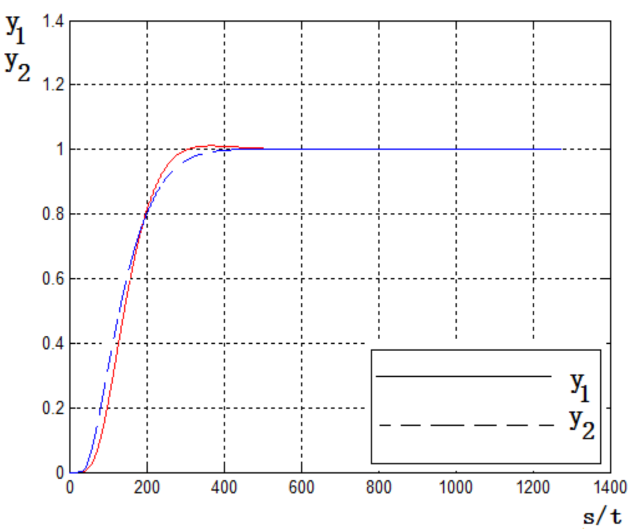

Figure 13 Operation results of PID control system and CSMFDNLM

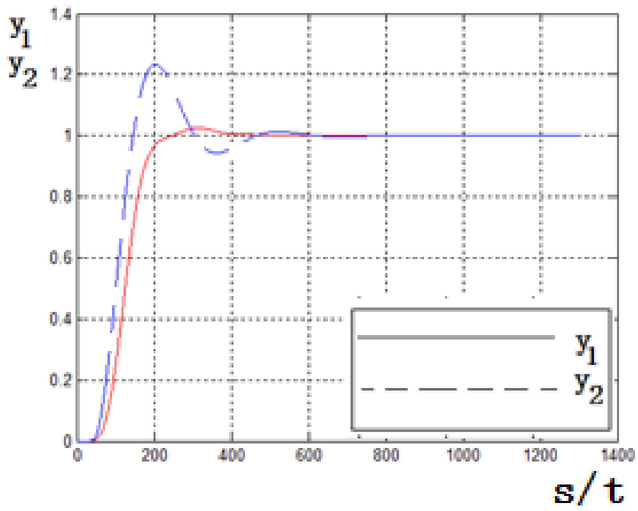

Figure 14 Operation results of PID control system and CSMFDNLM

6 Conclution

All parameters in these systems are only related to the desired transient process time T of system closed loop output. If the desired transient process time T of system output can't been defined by control engineer, then the process time $T = nt_s$ is set, where, $n \in (800\ 1500)$, t_s is the control period (or sample time period of DCS).

These examples of simulation and engineering application demonstrated the fine control quality of the PID controller with improved Smith predictor in time-invariance systems, and showed the fine control quality and robust performance of PIDC and CSMFDNLM in uncertain systems with model free design. The results in control theory and application demonstrated also the unity principle of Newton's laws of motion.

Design of control systems needs a mathematics model of controlled plant in general, however the model is not exactly obtained and the model is uncertain, these limits led to the poor control system performance. The paper aim at when a motion (/reaction) speed of controlled plant process (/body) is greatly slower than speed of light, its motion process can described by Newton's Laws of Motion, and these laws can be applied to the design and analysis the control systems quality.

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