

Equivalent Length of a Physical Pendulum: A Numerical Correction for the Simple Pendulum Formula



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Abstract: The simple pendulum formula is widely used in introductory physics laboratories to determine the gravitational acceleration. However, this formula assumes a point mass and a massless string, an idealization that fails when the pendulum bob has a finite size. This paper introduces the concept of an equivalent length such that the simple pendulum formula exactly reproduces the correct period of a physical pendulum consisting of a uniform spherical bob and a light string. Using elementary rigid-body dynamics, we derive an analytical expression for the correction factor, which depends only on the ratio of the string length to the bob radius. Numerical calculations over a wide range of this ratio show that the correction is below 0.5% when the string length is at least ten times the bob radius, but exceeds 4% when the ratio is two. A detailed lookup table and a classroom procedure are provided, allowing students to continue using the familiar simple pendulum equation while obtaining accurate values of gravitational acceleration. This work bridges the gap between idealized models and real physical systems without requiring advanced mathematics. The approach is purely computational and requires no laboratory equipment, making it suitable as a pre-lab exercise, a classroom demonstration, or an independent student project in numerical methods. By preserving the simple pendulum equation, the method reduces cognitive load and provides a natural bridge to more advanced topics such as moment of inertia and the parallel axis theorem. The results are exact within the stated assumptions and offer immediate practical utility for both high school and introductory university teachers.

Keywords: Simple Pendulum; Physical Pendulum; Equivalent Length; Numerical Correction; Moment of Inertia

DOI: [10.57237/j.edu.2026.05.002](https://doi.org/10.57237/j.edu.2026.05.002)

1 Introduction

The simple pendulum is one of the most fundamental systems in mechanics and has been used for centuries to measure the local acceleration due to gravity g [1, 2]. In its idealized form, a point mass is suspended by a massless, inextensible string of length L , and for small oscillations the period is $T = 2\pi\sqrt{L/g}$. In a typical classroom experiment, a real pendulum consists of a spherical bob of radius R and a string of length l (measured from the pivot to the

top of the bob). The total distance from the pivot to the centre of mass is $L = l + R$. Students often simply substitute this L into the simple pendulum formula, implicitly assuming that the bob is a point mass. When the bob size is not negligible compared to l , this approximation introduces a systematic error in the calculated g .

Recent studies by Oliveira have examined this error in detail. Oliveira (2022) [3] analysed the combined effects of

string mass, bob radius, and finite amplitude on the determination of g , and Oliveira (2026) [4] specifically investigated the influence of the R ratio using a large bob. These works demonstrate that for $R = 2$ the relative error in g can exceed 4%, and for $l = 0$ (bob pivoted at its surface) the error reaches 40%. Fulton and Gay (2024) [5] further explored the conditions under which a physical pendulum's period coincides with that of a simple pendulum, revealing a “magic” string length at which the two models agree. However, Oliveira's presentation focused on error analysis rather than providing a simple correction that allows students to keep the simple pendulum formula.

In recent years, physics education research has increasingly emphasized the importance of helping students navigate the gap between idealized models and real experimental systems [6, 7]. Studies have shown that when students encounter discrepancies between theoretical predictions and experimental results, they are more likely to engage in deeper inquiry and develop more robust conceptual understanding [8, 9]. This makes the development of practical correction tools particularly valuable for introductory physics instruction.

In this paper we take a different approach. Instead of discarding the simple pendulum model, we ask: What length L_{eff} would an ideal simple pendulum need to have the same period as the real physical pendulum? The answer yields an equivalent length that depends only on l and R . By using L_{eff} in place of $l + R$, students can obtain an accurate value of g while still working with the familiar equation $T = 2\pi\sqrt{L_{\text{eff}}/g}$. The derivation uses only basic rigid-body concepts (moment of inertia, parallel axis theorem) and leads to a compact analytical formula. Numerical calculations are then used to visualise the correction and to produce a practical table for classroom use.

This work is purely computational and requires no laboratory equipment, making it suitable as a pre-lab exercise, a classroom demonstration, or an independent student project in numerical methods [10, 11].

2 Theoretical Background

2.1 Period of a Physical Pendulum

Consider a uniform solid sphere of mass m and radius R suspended from a frictionless pivot by a light, inextensible string of length l . The string mass is assumed negligible compared to the bob mass — a condition easily satisfied in most classroom setups [3]. The distance from the pivot to

the centre of mass of the pendulum is $L = l + R$. The moment of inertia of the sphere about its own centre is $I_{\text{cm}} = \frac{2}{5}mR^2$. By the parallel axis theorem, the moment of inertia about the pivot is $I = I_{\text{cm}} + mL^2 = \frac{2}{5}mR^2 + m(l + R)^2$. For small angular displacements ($\theta_0 \ll 1$ rad), the period of the physical pendulum is $T_p = 2\pi\sqrt{\frac{I}{mgL}}$. Substituting I and L gives

$$T_p = 2\pi\sqrt{\frac{\frac{2}{5}R^2 + (l+R)^2}{g(l+R)}} \quad (1)$$

2.2 Definition of Equivalent Length

Let L_{eff} be the length of an ideal simple pendulum that has the same period as the physical pendulum:

$$T_p = 2\pi\sqrt{\frac{L_{\text{eff}}}{g}} \quad (2)$$

Equating equations (1) and (2) and squaring both sides yields

$$L_{\text{eff}} = \frac{\frac{2}{5}R^2 + (l+R)^2}{l+R} \quad (3)$$

Equation (3) is the desired expression for the equivalent length. It is always greater than $l + R$ because the additional term $\frac{2}{5}R^2$ is positive.

2.3 Dimensionless Correction Factor

It is convenient to introduce the dimensionless ratio $x = l/R$. Then $l = xR$ and $l + R = R(x + 1)$. Substituting into equation (3) gives $L_{\text{eff}} = \frac{\frac{2}{5}R^2 + R^2(x+1)^2}{R(x+1)} = R\frac{\frac{2}{5} + (x+1)^2}{x+1}$. The geometric length from pivot to centre of mass is $L = R(x + 1)$. Therefore the correction factor κ is

$$\kappa(x) = \frac{L_{\text{eff}}}{L} = \frac{\frac{2}{5} + (x+1)^2}{(x+1)^2} = 1 + \frac{2}{5(x+1)^2} \quad (4)$$

The relative error incurred if one uses the geometric length $L = l + R$ in the simple pendulum formula can also be expressed. Since $g_{\text{simple}} = 4\pi^2 L/T^2$ and the true value is $g_{\text{true}} = 4\pi^2 L_{\text{eff}}/T^2$, we have $\frac{g_{\text{true}}}{g_{\text{simple}}} = \kappa(x)$, and the relative error (definition: $\epsilon = (g_{\text{true}} - g_{\text{simple}})/g_{\text{true}}$) is

$$\epsilon(x) = 1 - \frac{1}{\kappa(x)} = 1 - \frac{1}{1 + \frac{2}{5(x+1)^2}} = \frac{2}{5(x+1)^2 + 2} \quad (5)$$

For large x , $\epsilon \approx 2/(5x^2)$, i.e., the error decays quadratically with l/R .

3 Numerical Calculations

All calculations were performed using a simple Python script. The steps were:

- 1) Define a range of $x = l/R$ from 0 to 100. For small x (0–10), a step of 0.05 was used to capture the rapid change; for larger x , a logarithmic spacing was adopted.
- 2) For each x compute $\kappa(x)$ from equation (4) and $\epsilon(x)$ from equation (5) (in percent: $\epsilon\% = 100 \cdot \epsilon$).
- 3) As an illustrative example, fix $R = 1.0$ cm and compute L_{eff} in cm.
- 4) Generate plots of κ and ϵ versus x using a semi-logarithmic scale for the horizontal axis.
- 5) Extract numerical values for a detailed lookup table (Table 1).

The code is straightforward and can be shared with students as an introductory programming exercise. Similar

computational approaches have been successfully integrated into undergraduate laboratories to enhance student engagement and conceptual understanding [12, 13]. No experimental data were used; all results follow directly from the analytical derivation.

4 Results and Discussion

4.1 Correction Factor κ and Relative Error ϵ

Figure 1 shows the correction factor $\kappa = L_{\text{eff}}/(l + R)$ as a function of l/R on a semi-logarithmic scale. When the string is much longer than the bob radius ($x \gg 1$), κ approaches 1 asymptotically, meaning the simple pendulum model is adequate. For instance, at $l/R = 10$, $\kappa = 1.00331$ (a correction of only 0.33%). At $l/R = 5$, $\kappa = 1.0111$ (1.11% correction). For $l/R = 2$, $\kappa = 1.0444$ (4.26% correction). In the extreme case $l = 0$ (the bob is pivoted directly on its surface), $x=0$ and $\kappa = 1.4$, i.e., the equivalent length is 40% longer than the geometric length.

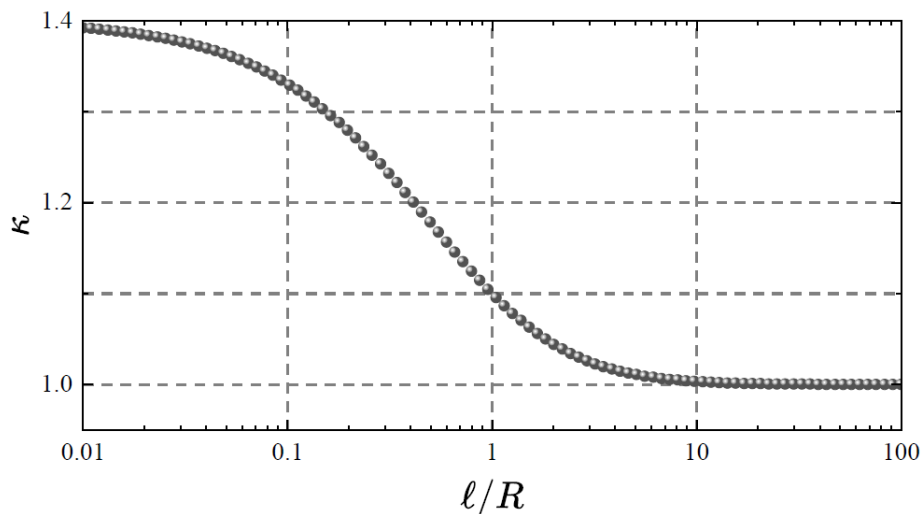


Figure 1 Correction factor $\kappa = L_{\text{eff}}/(l + R)$ as a function of the ratio l/R

Figure 2 displays the relative error ϵ (in percent) when the geometric length is used erroneously. For $l/R = 10$ the error is 0.329%; for $l/R = 3$ it is 2.44%; for $l/R = 1$ it reaches 9.09%. This error is systematic and can be eliminated by using L_{eff} instead of $l + R$.

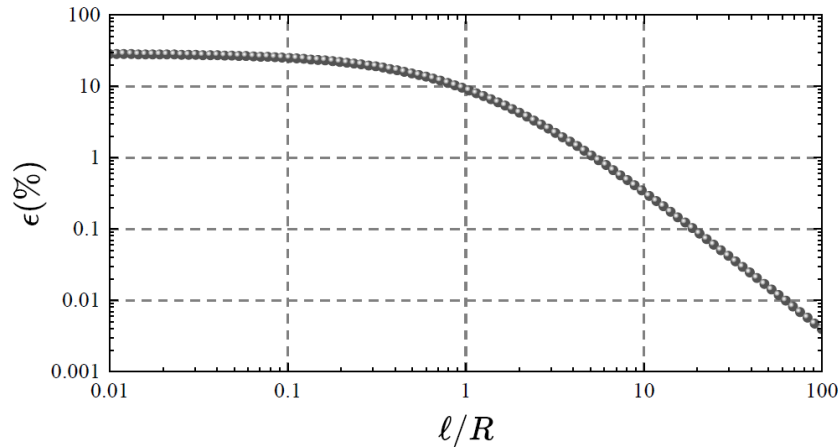


Figure 2 Relative error ϵ (in percent) when using the geometric length $L = l + R$ in the simple pendulum formula

4.2 Practical Lookup Table for Classroom Use

Table 1 provides a comprehensive set of values for easy

classroom application. The table assumes a bob radius $R = 1$ cm, but because κ depends only on the ratio l/R , the values of l , $L = l + R$, and L_{eff} scale linearly with R . For a bob of different radius, multiply the tabulated lengths by the actual R (in cm). The correction factor κ and the relative error $\epsilon(\%)$ are universal.

Table 1 Correction parameters for a pendulum with a spherical bob (radius $R = 1$ cm). For other radii, lengths scale linearly; κ and ϵ remain unchanged

l/R	l (cm)	$L = l + R$ (cm)	L_{eff} (cm)	$\kappa = L_{\text{eff}}/L$	ϵ (%)
0.0	0.00	1.00	1.4000	1.4000	28.57
0.5	0.50	1.50	1.7667	1.1778	15.09
1.0	1.00	2.00	2.2000	1.1000	9.091
1.5	1.50	2.50	2.6600	1.0640	6.015
2.0	2.00	3.00	3.1333	1.0444	4.255
2.5	2.50	3.50	3.6143	1.0327	3.163
3.0	3.00	4.00	4.1000	1.0250	2.439
4.0	4.00	5.00	5.0800	1.0160	1.575
5.0	5.00	6.00	6.0667	1.0111	1.099
6.0	6.00	7.00	7.0571	1.0082	0.810
7.0	7.00	8.00	8.0500	1.00625	0.621
8.0	8.00	9.00	9.0444	1.00494	0.492
9.0	9.00	10.00	10.0400	1.00400	0.398
10.0	10.00	11.00	11.0364	1.00331	0.329
12.0	12.00	13.00	13.0308	1.00237	0.236
15.0	15.00	16.00	16.0250	1.00156	0.156
20.0	20.00	21.00	21.0190	1.00091	0.0907
30.0	30.00	31.00	31.0129	1.00042	0.0416
50.0	50.00	51.00	51.0078	1.00015	0.0154
100.0	100.00	101.00	101.0040	1.00004	0.0039

4.3 Classroom Procedure

From equation (4), the true gravitational acceleration is obtained as

$$g_{\text{true}} = \frac{4\pi^2 L_{\text{eff}}}{T^2} = \frac{4\pi^2 (l+R)}{T^2} \left(1 + \frac{2}{5(l/R+1)^2}\right) \quad (6)$$

Thus, a student needs only:

- 1) Measure the string length l (from pivot to top of bob) with a ruler.
- 2) Measure the bob radius R (e.g., with a caliper).
- 3) Measure the oscillation period T for small amplitudes ($\theta_0 < 10^\circ$).
- 4) Plug these numbers into equation (6) using a calculator, or look up the correction factor from Table 1.

4.4 Discussion: Pedagogical Value and Limitations

The concept of equivalent length has two major pedagogical advantages. First, it preserves the familiar form of the simple pendulum equation, reducing cognitive load while still correcting for finite bob size. Second, it provides a natural bridge to more advanced topics: after using L_{eff} , a teacher can ask "Why is L_{eff} larger than $l + R$?" leading to a discussion of moment of inertia and the parallel axis theorem. This scaffolding approach is particularly effective in introductory courses, as it aligns with research showing that students benefit from explicit connections between mathematical formalism and physical meaning [14]. Furthermore, recent studies have demonstrated that video-based analysis and computer-assisted experiments can significantly improve students' understanding of pendulum motion and gravity measurement [15, 16].

Our analysis assumes a uniform solid sphere and a massless string. In many classroom pendulums, the bob may be hollow (e.g., a ping-pong ball) or the string may have non-negligible mass. Oliveira [3] showed that string mass causes a negative error (tending to reduce the period), which partially cancels the positive error due to bob radius. For a hollow sphere with moment of inertia $I_{\text{cm}} = \frac{2}{3}mR^2$, the correction factor becomes $\kappa = 1 + \frac{2}{3(1+l/R)^2}$. The present method can be easily adapted to such cases by changing the factor $2/5$ to the appropriate coefficient.

Another limitation is the small-angle approximation. For amplitudes larger than 10° , the period increases further due to the elliptic integral correction [6]. Teachers should instruct students to keep the angular amplitude below 10° so that the amplitude error remains below 0.1% and can be neglected.

Finally, all calculations in this paper are analytical; the numerical results merely illustrate the formulae. Therefore, the conclusions are exact within the stated assumptions and do not depend on measurement uncertainties. This makes the work a reliable reference for both teachers and students.

5 Conclusion

We have introduced the concept of an equivalent length L_{eff} for a physical pendulum consisting of a uniform spherical bob and a light string. The equivalent length is defined so that the simple pendulum formula $T = 2\pi\sqrt{L_{\text{eff}}/g}$ gives the exact period. The analytical result

$$L_{\text{eff}} = (l + R) \left[1 + \frac{2}{5(1+l/R)^2} \right]$$

is derived using only elementary rigid-body dynamics. Numerical calculations show that the correction is negligible ($\lesssim 0.5\%$) when $l/R \geq 10$, but becomes significant (up to 40%) for very short strings.

A detailed lookup table (Table 1) and a simple classroom procedure are provided. Students need only measure l , R , and T , then compute g via equation (6) or use the table to obtain L_{eff} . This approach allows instructors to retain the simple pendulum model as the primary teaching tool while eliminating a common systematic error. The work is purely computational, requires no special equipment, and can be implemented as a short programming exercise or a pre-lab activity. It bridges the gap between idealized models and real-world experiments, fostering deeper understanding of both rotational inertia and practical error analysis.

Author Contributions

Wen-Yuan Wang: Conceptualization, Methodology, Software, Formal Analysis, Investigation, Writing – original draft, Writing – original draft, Visualization

Hong-Juan Meng: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Project administration, Writing – review & editing

Data Availability Statement

All data generated in this study are derived directly from the analytical formulae presented in the paper. The Python script used for the numerical calculations and for generating the table is available from the corresponding author upon reasonable request.

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