A Multi-Stage Stochastic Programming Model with CVaR for Aviation Spare Parts Inventory Optimization

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Abstract: In aviation Maintenance, Repair, and Overhaul (MRO) systems, spare parts inventory management faces highly stochastic demand, significant tail risks, supply delays, and complex contractual constraints, making it difficult for traditional deterministic or risk-neutral models to balance high service levels with low costs. This paper proposes a multi-stage stochastic programming inventory optimization model incorporating Conditional Value-at-Risk (CVaR) to enhance economic efficiency and robustness. The model employs a scenario generation framework based on moment matching and entropically regularized optimal transport that preserves inter-part correlations through Pearson coefficients and efficiently reduces scenario trees via Sinkhorn distance, accurately capturing demand volatility and extreme tail events. It further incorporates non-anticipativity constraints in a multi-stage mixed-integer stochastic program, along with rolling-horizon mechanisms, tiered pricing, and contract restrictions to enable dynamic ordering decisions under sequentially revealed information. A Mean-CVaR optimization criterion with a tunable risk aversion parameter is introduced to balance expected cost against extreme losses. Numerical experiments on typical spare parts from a commercial airline's B737 fleet show that the risk-averse model modestly increases expected cost relative to the risk-neutral version but substantially reduces tail risk. Stochastic strategies outperform deterministic approaches, with value-of-stochastic-solution metrics confirming clear advantages under uncertainty, while model stability and robustness are validated through in-sample and out-of-sample testing. This study offers airlines a practical framework for developing cost-effective and safe spare parts inventory policies in uncertain environments.

Keywords: Multi-stage Stochastic Programming; CVaR; Aviation Spare Parts Inventory; Moment Matching

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1 Introduction

In aviation MRO systems, spare parts inventory management has long been at the center of the trade-off between stringent high service level requirements and the pursuit of low operational costs. As a critical component in ensuring fleet airworthiness, spare parts inventory not only ties up substantial working capital for airlines but also directly impacts flight punctuality and safety. In practice, however, spare parts management departments face increasingly severe challenges. On one hand, demand for spare parts is

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heavily influenced by fleet composition, maintenance policies, route networks, and seasonal variations (e.g., differences between winter and summer weather), exhibiting pronounced stochasticity and non-stationarity. Traditional "experience-based extrapolation" approaches relying on historical averages struggle to capture the "long-tail" risks inherent in demand distributions. On the other hand, the inherent complexity of aviation supply chains—such as supplier capacity constraints, tiered pricing terms, and lengthy order lead times—results in significant lags in supply response, leading to severe spatiotemporal mismatches between supply and demand. Most critically, the aviation industry's extreme sensitivity to safety dictates a strong risk-averse orientation among decision-makers, where the costs of stockouts leading to aircraft-on-ground (AOG) events far exceed holding costs, often prompting managers to maintain excessive safety stock and thereby reducing capital efficiency.

Extensive research has addressed these uncertainty management challenges. Early inventory management studies were primarily based on deterministic assumptions, with the classic Economic Order Quantity (EOQ) model proposed by Harris laying the foundation for the field [1]. Subsequent works by Dye and Liao, among others, extended this framework by incorporating deterioration rates and delayed payments, further enriching deterministic models [2, 3]. However, in complex engineering environments like civil aviation MRO, where demand volatility and supply delays are prominent, deterministic models prove inadequate. Consequently, research has shifted toward stochastic inventory models. Pashapour et al. developed an approximate dynamic programming approach to optimize inventory and production in remanufacture-to-order systems [4], while Reddy and Kumar integrated capacity and inventory decisions using two-stage stochastic programming [5]. Although Taleizadeh and others extended single-period multi-product models to multi-period stochastic inventory control with chance constraints, most of these studies adopt risk-neutral assumptions and overlook decision-makers' sensitivity to low-probability, high-impact extreme events, resulting in strategies that lack robustness in high-risk real-world settings [6].

To address the limitations of risk-neutral models, incorporating risk measures into inventory decisions has become an inevitable trend. Early risk-aware approaches often relied on utility functions or mean-variance frameworks, but the former is subjective, while the latter struggles with asymmetric long-tail risks [7, 8]. With the introduction of CVaR by Rockafellar and Uryasev, its superior mathematical properties have made it a dominant risk measure. In recent years, integrating CVaR into stochastic programming has demonstrated clear advantages in supply chain management. For instance, Xu and Chen used CVaR to quantify risk and maximize loss-averse utility for optimal retailer ordering [11]; Xu et al. applied CVaR to model procurement risk in newsvendor problems under mismatch cost minimization [12]. Hu et al. incorporated CVaR in two-stage models to mitigate potential risks [13], while multi-stage risk-averse models have proven effective in balancing expected costs and tail risks in complex systems such as energy planning and disaster management [14, 15]. Nevertheless, existing studies predominantly focus on two-stage decisions or static single-product risk control. Research on multi-stage risk-averse models tailored to aviation spare parts—with their multi-period rolling-horizon characteristics, inter-part correlations, and complex contractual constraints—remains relatively scarce.

To address these challenges and research gaps, this paper proposes a multi-stage stochastic programming inventory model for aviation spare parts that explicitly accounts for CVaR and supply-demand uncertainties. The main contributions are as follows:

First, a high-fidelity scenario generation framework is developed based on moment matching and entropically regularized optimal transport. This approach explicitly incorporates Pearson correlation coefficients to preserve inter-part failure dependencies and employs the Sinkhorn distance algorithm for efficient reduction from large Monte Carlo samples to a discrete scenario tree, achieving high statistical fidelity while significantly reducing computational complexity.

Second, a multi-stage mixed-integer stochastic programming (MIP) model with stage-wise non-anticipativity constraints is formulated. By overcoming the limitations of traditional two-stage models, it incorporates rolling-horizon mechanisms to dynamically adjust quarterly ordering strategies and integrates realistic tiered pricing and contractual constraints.

Third, a Mean-CVaR optimization criterion is introduced. By quantifying decision-makers' risk aversion, it achieves an optimal balance between expected costs and tail risks, providing airlines with theoretical support and quantitative tools for developing inventory strategies that are both robust and economically efficient.

2 Scenario Tree Construction via Moment Matching and Entropy-Regularized Optimal Transport

To accurately characterize the time-varying nature, volatility, and coupling relationships of aviation spare parts demand, this paper proposes a three-layer scenario generation framework. First, distributions are established based on fleet operational characteristics and seasonal factors. Second, initial joint scenarios are generated using a moment matching method that incorporates Pearson correlation coefficients. Finally, a multi-stage scenario tree is constructed utilizing the Fast Forward Selection (FFS) algorithm combined with entropy regularization.

2.1 Stochastic Demand Distribution Specification and Monte Carlo Simulation

Taking the B737 fleet (137 aircraft) of a specific airline as the subject of study, and considering the distinct failure mechanisms of different spare parts, this paper employs two types of count data distributions to describe quarterly demand rates:

(1) Hydraulic Pump Assembly

For spare parts where failures occur relatively independently with mild fluctuations, the Poisson distribution is adopted. Furthermore, time-varying mean parameters are calibrated by incorporating seasonal operational characteristics (e.g., high utilization during the summer peak season).

Let the annual mean demand be denoted by $\lambda^{\text{year}} = 33$. We introduce quarterly demand weights w_t, satisfying the condition.

$$\sum_{t=1}^{4} w_t = 1, \quad w_t \geq 0 .$$

Accordingly, the quarterly demand intensity is defined as

$$\lambda_t = \lambda^{\text{year}} \cdot w_t .$$

In light of the peak and off-peak patterns of airline operations and variations in quarterly mission volumes, the summer transport season is characterized by higher flight density and increased take-off and landing cycles. This leads to more concentrated component wear and scheduled replacements, yielding the intensity parameters for the four quarters as $\lambda_1 = 6$, $\lambda_2 = 8$, $\lambda_3 = 12$, $\lambda_4 = 7$.

Based on the aforementioned parameter specifications, a large number of raw demand sample sequences for each quarter are generated via Monte Carlo simulation, serving as the foundational input for subsequent scenario generation.

(2) Wheel Brake Assembly

For spare parts significantly affected by environmental factors (e.g., corrosion from de-icing fluid in winter, heat accumulation in summer) and exhibiting "over-dispersion" characteristics, the simple Poisson assumption is insufficient to capture their long-tail risks. This paper introduces the NB2 parameterized Negative Binomial distribution, which allows the variance to significantly exceed the mean, thereby effectively characterizing the heavy-tail features of the demand.

Table 1 Wheel Brake Assembly Quarterly Demand Distribution Specification

Quarter	**Seasonal Characteristics**	**Mean**	**Variance**	**Dispersion Ratio**	**NB2**
Quarter 1	Deep Winter/Early Spring	104	161	1.55	189.75
Quarter 2	Spring/Early Summer	115	124	1.09	1469.44
Quarter 3	Peak Summer Season	132	149	1.13	1024.94
Quarter 4	Autumn/Early Winter	101	135	1.34	300.02

2.2 Scenario Generation via Moment Matching Considering Correlations

Traditional univariate moment matching typically focuses solely on mean, variance, skewness, and kurtosis, often neglecting the dependency structure among spare parts demands. To ensure that the generated scenario set preserves the joint distribution characteristics of historical data, this paper explicitly incorporates the Pearson Correlation Coefficient into the feature matching objective function.

For the two-dimensional joint scenario generation problem addressed in this paper, the specific objective function and constraints are formulated as follows:

$$\min_{x,p} \sum_{k \in Q} w_k \left(f_k(x,p) - S_k^* \right)^2$$
$$s.t. \sum_{i=1}^{N} p_i = 1$$
$$p_i \geq 0, \quad \forall i = 1, \ldots, N.$$

Where:

x represents the set of two-dimensional demand values in each scenario; p is a column vector composed of the probabilities of occurrence for each scenario; N denotes the specified total number of scenarios; Q is the set of statistical features; S_k^* represents the target value of the k -th statistical feature derived from the reference sample set; $f_k(x,p)$ is the mathematical expression for statistical feature k in terms of scenario values and probabilities p . In this context, this includes not only the mean, variance, skewness, and kurtosis of the two respective spare parts but also the Pearson correlation coefficient, which describes the degree of association between them.

Regarding the introduced correlation coefficient feature, its calculation formula within the discrete scenario set is defined as:

$$f_{corr}(x,p) = \frac{\sum_{i=1}^{N} p_i \left(x_{i,1} - \mu_1 \right)\left(x_{i,2} - \mu_2 \right)}{\sigma_1 \sigma_2}$$

Where $x_{i,1}$ and $x_{i,2}$ denote the demand values of the two types of spare parts under the i -th scenario, respectively; μ and σ denote the mean and standard deviation of the corresponding spare parts under the current scenario distribution. w_k represents the weight of each statistical feature k . To eliminate the substantial differences in numerical magnitude across moments of different orders (such as between kurtosis and mean) and to prevent high-order moments from dominating the optimization direction, this paper sets the weight as the inverse square of the absolute value of the target statistic, that is,

$$w_k = 1 / \max\left(\left| S_k^* \right|, 1 \right)^2$$

2.3 Scenario Reduction Based on Entropy-Regularized Optimal Transport

In multi-stage stochastic programming, if a scenario tree is constructed using a full enumeration approach, the total number of scenarios grows exponentially with the number of stages, rendering the subsequent MIP models difficult to solve. Consequently, effective reduction of the scenario set is requisite.

Traditional scenario reduction algorithms are typically based on the Kantorovich distance (Wasserstein distance) and a "nearest neighbor" hard assignment strategy, which often leads to discontinuity in probability distributions and the loss of local information. To address this, this paper proposes an improved FFS algorithm that integrates Entropy-Regularized Optimal Transport.

This method introduces a negative entropy penalty term into the original Wasserstein distance objective function and utilizes the Sinkhorn distance to replace the traditional Euclidean distance.

(1) Introduction of Recursive Distance Update

To address the issue of repetitive distance matrix calculations in the standard forward selection method, the algorithm in this paper utilizes the recursive formula proposed by Heitsch & Römisch [16]. Instead of rescanning all distances each time, an auxiliary vector $D_{min}^{(k)}$ is maintained to record the shortest distance from each original scenario to the currently selected set. When a new point is added, it is only necessary to update.

$$D_{min}^{(k+1)}(i) = \min(D_{min}^{(k)}(i), d(\xi_i, \xi_{new})) .$$

This improvement boosts algorithmic efficiency by several orders of magnitude.

(2) Probability Re-estimation Based on Entropy Regularization

Traditional probability assignment typically adopts a "hard assignment" strategy based on Voronoi partitions (i.e., the nearest neighbor principle). This method often leads to discontinuity in probability distributions and the loss of local feature information when processing samples located at cluster boundaries. To overcome this defect, this paper introduces the entropy regularization method from Optimal Transport theory.

This method adds a negative entropy penalty term $H(\pi)$ to the original Wasserstein distance objective function, transforming the originally complex linear programming problem into a convex optimization problem. According to Sinkhorn theory, the optimal transport plan Π for this problem possesses a specific exponential structure. For the specific scenario in this paper where the "target distribution weights are unknown" (i.e., the second marginal is free), a closed-form solution for the optimal weights can be derived, thereby avoiding the cyclic iteration process of the traditional Sinkhorn algorithm.

The specific calculation formula is as follows: Construct the kernel matrix $K_{ij} = \exp(-d(\xi_i, \xi_j)/\gamma)$, where γ is the regularization coefficient. The optimal re-estimated probability q_j for the retained scenario j can be directly obtained from the following equation:

$$q_j = \sum_{i=1}^{N} p_i \frac{K_{ij}}{\sum_{m \in J} K_{im}} = \sum_{i=1}^{N} p_i \frac{\exp\left(-d\left(\xi_i, \xi_j\right)/\gamma\right)}{\sum_{m \subset J} \exp\left(-d\left(\xi_i, \xi_m\right)/\gamma\right)}$$

This formula is essentially a variant of the Softmax function. When $\gamma \to 0$, this weight assignment converges to the traditional nearest-neighbor hard assignment; whereas under an appropriate value of γ, the method utilizes Sinkhorn geometric properties to realize a smooth transition of probability mass, thereby enabling the reduced scenario set to more robustly characterize the distribution shape of the original stochastic process.

This improvement not only leverages the recursive distance update formula to boost computational efficiency by several orders of magnitude, but also strengthens the robustness of the reduced scenario set in capturing the tail extreme value characteristics of the original distribution.

2.4 Scenario Generation Results

By applying the aforementioned methods, this study ultimately constructs a multi-stage scenario tree comprising 80 representative scenarios, thereby establishing a solid data foundation for solving the multi-stage stochastic programming model. Partial retained scenario sets from the demand forecasts, along with their corresponding probabilities, are presented in Tables 2 and 3.

Table 2 Hydraulic Pump Assembly Demand Retained Scenario Set (Partial)

Scenario	**Probability**	Q_1 **Demand**	Q_2 **Demand**	Q_3 **Demand**	Q_4 **Demand**
1	0.079296419	7	9	14	7
2	0.01288561	7	9	14	12
3	0.078350939	7	9	9	7
4	0.01273197	7	9	9	12
5	0.083746857	7	6	14	7
74	0.000917697	5	9	14	12
77	0.003430426	5	14	14	7
78	0.000557442	5	14	14	12
79	0.003389523	5	14	9	7
80	0.000550795	5	14	9	12

Table 3 Wheel Brake Assembly Demand Retained Scenario Set (Partial)

Scenario	**Probability**	Q_1 **Demand**	Q_2 **Demand**	Q_3 **Demand**	Q_4 **Demand**
1	0.079296419	99	123	138	97
2	0.01288561	99	123	138	107
3	0.078350939	99	123	126	97
4	0.01273197	99	123	126	107
5	0.083746857	99	117	138	97
74	0.000917697	123	104	138	107
77	0.003430426	123	113	138	97
78	0.000557442	123	113	138	107
79	0.003389523	123	113	126	97
80	0.000550795	123	113	126	107

3 A Multi-Stage Stochastic Programming Inventory Model with CVaR

3.1 Problem Description and Basic Assumptions

This study focuses on the dynamic inventory optimization problem of aircraft spare parts for airlines under a multi-period, multi-scenario environment. The decision-making process follows the Rolling Horizon mechanism: At the initial stage, the decision maker needs to sign long-term procurement contracts with suppliers based on the annual flight plan, determining the contract base quantity and the corresponding tiered pricing levels. In subsequent quarters, as spare part failures and demand information are gradually revealed, the decision maker makes recourse orders based on the current inventory status.

In order to transform the above engineering problem into a solvable mathematical model while reflecting the core business characteristics under the premise of ensuring computational tractability, the following basic assumptions are made in this paper:

Assumption 1: Uncertainty Representation. The uncertainty of spare part demand and related stochastic parameters within the planning horizon is fully described by the discrete scenario tree generated in Chapter 2. Each node in the scenario tree represents a system state, each path represents a possible future evolution scenario, and the probabilities of all scenarios are known.

Assumption 2: Non-anticipativity. Inventory decisions are made in stages. At the beginning of each stage, the decision maker can only make decisions based on information available up to the current time and cannot foresee which specific scenario will occur in the future. That is, the model must satisfy the non-anticipativity constraints (NAC). ensuring that decision variables at the same moment and under the same history path must be equal, and no "cheating" is allowed by using future information.

Assumption 3: Supply and Contract Constraints. The ordering quantity of critical aircraft spare parts must satisfy integer constraints. Considering the supply chain lead time, it is assumed that regular orders have a fixed lead time of one period. Meanwhile, the procurement price follows an all-unit discount mechanism based on the total annual order quantity.

Assumption 4: Inventory and Shortage Mechanism. In actual engineering practice, since shortage events are usually accompanied by early warning mechanisms, the shortage cost in the model is calculated based on the expenses required for emergency ordering, rather than representing the actual losses caused by production stoppage.

3.2 Multi-Stage Stochastic Programming Model with Recourse

This study addresses the quarterly rolling decision problem for spare parts ordering and inventory control by formulating a multi-stage stochastic programming model that incorporates stage-wise non-anticipativity constraints. Unlike traditional two-stage models, which implicitly assume perfect foresight of all uncertainties, the proposed model more accurately represents the real-world setting in which demand is revealed period by period and information is updated incrementally. Through a dynamic process of "decision — random realization — state update — re-decision," the model enforces decision consistency within the same historical information set, thereby better capturing the operational characteristics of spare parts management.

Consider a finite scenario set. Let Equation S denote the complete scenario set, $|S|$ the number of scenarios, ξ^s the realization of random variables in scenario s, and p^s the corresponding probability. The deterministic equivalent formulation of the multi-stage stochastic program is then given by

$$\min \sum_{s\in S} p^s \sum_{t=1}^{T} c_t\left(x_t^s, \xi_t^s\right)$$

where x_t^s represents the decision variables at stage t under scenario t. The information set at stage t is defined by the historical random variable realizations observed prior to the decision at stage t. If scenarios s and s' are indistinguishable under this information set, the stage-wise non-anticipativity constraints $x_t^s = x_t^{s'}$ $t = 2,\dots,T$ must hold. These constraints ensure decision consistency conditional on the same historical information and prevent the use of future information in a clairvoyant manner.

In line with the quarterly rolling nature of the maintenance spare parts inventory problem studied here, each quarter is treated as a stage. Strategic decisions—such as the annual contract and initial orders made at the beginning of the first quarter—are treated as first-stage decisions and are scenario-independent. Ordering decisions at the beginning of quarters 2–4, along with other inventory-related decisions, are modeled as subsequent recourse decisions that depend on observed historical demand. Demand is realized and observed within each quarter; state variables (inventory, shortages, maintenance status, etc.) are updated according to inventory balance equations and capacity constraints, and the updated information is carried forward to the next quarter's decision process. This multi-stage stochastic programming framework enables the development of implementable rolling ordering and inventory control policies under sequentially revealed uncertain demand, thereby providing a solid modeling foundation for reducing costs and improving service levels in maintenance spare parts inventory systems.

3.3 Formulation of the Risk-Neutral Multi-Stage Stochastic Programming Inventory Model

Table 4 Sets and Indices for the Stochastic Programming Inventory Model

i	spare part categories	$i=1,\dots,I$
j	suppliers	$j=1,\dots,J$
t	time periods (quarters)	$t=1,\dots,T$
m	order quantity batch intervals	$m=1,\dots,M$
s	demand scenarios	$s=1,\dots,S$

Table 5 Parameters for the Stochastic Programming Inventory Model

Symbol	Description
$D_{i,t,s}$	Demand for spare part i in quarter t under scenario s
P_s	Probability of scenario s occurring
P^r	Repair success rate of spare part i (unrepaired items are scrapped)
P_k	Procurement price intervals (batch quantity ranges)
$c_{i,j}$	Unit procurement price for spare part i from supplier j, based on contracted order volume base
$\alpha_{i,j}$	Contract unit price for spare part i from supplier j
C_i^{repair}	Unit repair cost for spare part i
h_i	Unit holding cost for serviceable spare part i
h_i^u	Unit holding cost for unserviceable (awaiting repair) spare part i
C_i^{install}	Unit installation/replacement cost for spare part i
S_i	Unit shortage penalty cost for spare part i
$c_{tr_{i,j,m,s}}$	Transportation cost when ordering spare part i from supplier j in batch interval m under scenario s
$\overline{K}$	Total inventory capacity
$pd_{i,j,t}$	Defective rate of spare part i supplied by supplier j in quarter t
$pl_{i,j,t}$	Delay rate of spare part i supplied by supplier j in quarter t
$tol_{q_{i,j,t}}$	Quality tolerance threshold for spare part i from supplier j in quarter t
$tol_{l_{i,j,t}}$	On-time delivery tolerance threshold for spare part i from supplier j in quarter t
$\mathrm{U}_{i,j,t,m}$	Upper bound on order quantity in the batch interval m for spare part i from supplier j in quarter t
$\mathrm{U}^k{}_{i,j,k}$	Upper bound on the contracted order volume base in the batch interval k for spare part i from supplier j
$k_{i,t,s}$	Delivery lead time matrix for spare part i in quarter t under scenario s

Table 6 Decision Variables for the Stochastic Programming Inventory Model

Symbol	**Description**
$Q_{i,j,t,s}$	Order quantity of spare part i from supplier j in quarter t under scenario s
$\delta_{i,j,t,m,s}$	Binary variable: equals 1 if the order quantity of spare part i from supplier j in quarter t under scenario s falls in the m-th batch interval
$\omega_{i,j,k}$	Binary variable: equals 1 if the contracted order volume base for spare part i from supplier j falls in the k-th interval
$b_{i,j,t,m,s}$	Actual order quantity of spare part i from supplier j in quarter t under scenario s that falls in the m-th batch interval
$Q^{u}{}_{i,t,s}$	Number of successfully repaired spare parts i in quarter t under scenario s
$r_{i,t,s}$	End-of-quarter inventory of serviceable spare part i in quarter t under scenario s
$u^{r}{}_{i,t,s}$	End-of-quarter inventory of unserviceable (awaiting repair) spare part i in quarter t under scenario s
$x^{r}{}_{i,t,s}$	Number of serviceable spare parts i actually used for aircraft maintenance in quarter t under scenario s
$l_{i,t,s}$	Unmet (shortage) demand for spare part i in quarter t under scenario s
$X_{i,j}$	Annual contracted order quantity of spare part i from supplier j (first-stage decision)

Table 7 Auxiliary Variables for the Stochastic Programming Inventory Model

Symbol	**Description**
$\mathrm{R}^{f}{}_{i,j,t,s}$	Number of defective spare parts i received from supplier j in quarter t under scenario s
$\mathrm{R}^{l}{}_{i,j,t,s}$	Number of delayed spare parts i received from supplier j in quarter t under scenario s
$R^{\mathrm{lf}}{}_{i,j,t,s}$	Total number of invalid spare parts i (defective or delayed) received from supplier j in quarter t under scenario s
$y_{i,t,s}$	Total number of spare parts i that actually arrive in quarter t under scenario s

The objective function is expressed as follows:

$$\min Z=\sum_{i=1}^{I}\sum_{j=1}^{J}\alpha_{i,j}X_{i,j}+\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{j=1}^{J}P_s*\left(c_{i,j}Q_{i,j,t,s}+\sum_{m=1}^{M}c_{tr_{i,j,m,s}}*\delta_{i,j,t,m,s}\right)$$
$$+\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{i=1}^{I}P_s*\left(C_i^{repair}*u^{r}{}_{i,t-1,s}+h_i*r_{i,t,s}+h_i^{u}*u^{r}{}_{i,t,s}+S_i*l_{i,t,s}+C_i^{install}*x^{r}{}_{i,t,s}\right)$$

For the aviation spare parts inventory management problem, grounded in engineering practice and aligned with cost control objectives, the objective function of the proposed model comprehensively captures seven primary cost components. The first is the contract reservation cost, which represents the payment made in the first stage to secure supplier capacity. Next are the procurement cost and transportation cost, which respectively denote the spare parts acquisition expenses in each stage—calculated based on price discounts—and the logistics delivery expenses determined by order batches. In the maintenance and support operations, the model includes repair cost and installation cost: the former covers variable expenses incurred during the repair of failed parts, while the latter accounts for labor and ancillary expenses associated with installing serviceable spare parts onto aircraft. Finally, for inventory and risk management, the model incorporates inventory holding cost and shortage penalty cost. The holding cost reflects the capital tied up in both serviceable spare parts and parts awaiting repair, whereas the shortage penalty cost is calibrated based on the high premium of emergency replenishment, thereby capturing the potential operational risks to flight schedules resulting from supply shortages.

Constraints:

$$\sum_{t=1}^{T}Q_{i,j,t,s}=X_{i,j}\qquad\forall i\in I,j\in J,s\in S$$

$$Q_{i,j,t,s}=Q_{i,j,t,s'}\quad\forall\left(s,s'\right)\in\mathcal{S}_t\quad z_{t,s}=z_{t,s'}\quad\forall\left(s,s'\right)\in\mathcal{S}_t$$

$$pd_{i,j,t} * Q_{i,j,t,s} \le tol^{q}_{i,j,t,s} \qquad \forall i \in I, j \in J, t \in T, s \in S$$

$$R^{f}{}_{i,j,t,s} = pd_{i,j,t} * Q_{i.j,t,s} \qquad \forall i \in I, j \in J, t \in T, s \in S$$

$$k_{i,t,s} = 1 \qquad \forall i \in I, t \in T, s \in S$$

$$R^{l}{}_{i,j,t,s} = pl_{i,j,t} * Q_{i,j,t,s} \qquad \forall i \in I, j \in J, t \in T, s \in S$$

$$R^{lf}{}_{i,j,t,s} = R^{l}{}_{i,j,t,s} + R^{f}{}_{i,j,t,s} \qquad \forall i \in I, j \in J, t \in T, s \in S$$

$$\sum_{j=1}^{J} (Q_{i,j,t,s} - R^{lf}{}_{i,j,t,s} + R^{l}{}_{i,j,t-1,s} * k_{i,t,s}) = y_{i,t,s} \qquad \forall i \in I, t \in T, s \in S$$

$$R^{l}{}_{i,j,0,s} = 0 \qquad R^{lf}{}_{i,j,0,s} = 0 \qquad \forall i \in I, j \in J, s \in S$$

$$Q_{i,j,t,s} = \sum_{m=1}^{M} b_{i,j,t,m,s} \qquad \forall i \in I, j \in J, t \in T, m \in M, s \in S$$

$$L_{i,j,t,m} * \delta_{i,j,t,m,s} \le b_{i,j,t,m,s} \le U_{i,j,t,m} * \delta_{i,j,t,m,s} \qquad \forall i \in I, j \in J, t \in T, m \in M, s \in S$$

$$\sum_{m=1}^{M} \delta_{i,j,t,m,s} = 1$$

$$\sum_{i=1}^{I} (r_{i,t,s} + u^{r}{}_{i,t,s}) \le \bar{K} \qquad \forall t \in T, s \in S$$

$$r_{i,t,s} = r_{i,t-1,s} + y_{i,t,s} + p^{r} u^{r}{}_{i,t-1,s} - x^{r}{}_{i,t,s} \qquad \forall i \in I, t \in T, s \in S$$

$$x^{r}_{i,t,s} \le r_{i,t-1,s} + y_{i,t,s} + p^{r} u^{r}{}_{i,t-1,s} \qquad \forall i \in I, t \in T, s \in S$$

$$u^{r}{}_{i,t,s} = x^{r}{}_{i,t,s} \qquad \forall i \in I, t \in T, s \in S$$

$$D_{i,t,s} = x^{r}{}_{i,t,s} + l_{i,t,s} \qquad \forall i \in I, t \in T, s \in S$$

$$Q^{u}{}_{i,t,s} = p^{r} * u^{r}{}_{i,t-1,s} \qquad \forall i \in I, t \in T, s \in S$$

$$u^{r}{}_{i,0,s} = 0 \qquad \forall i \in I, s \in S$$

$$\sum_{k=1}^{K} \omega_{i,j,k} \le 1 \qquad \forall i \in I, j \in J$$

$$\sum_{k=1}^{K} L^{k}{}_{i,j,k} \omega_{i,j,k} \le X_{i,j} \le \sum_{k=1}^{K} U^{k}{}_{i,j,k} \omega_{i,j,k} \qquad \forall i \in I, j \in J$$

$$c_{i,j} = \sum_{k=1}^{K} p_k \omega_{i,j,k} \qquad \forall i \in I, j \in J$$

$$R^{lf}{}_{i,j,t,s}, b_{i,j,t,m,s}, x^{r}{}_{i,t,s}, l_{i,t,s}, r_{i,t,s}, u^{r}{}_{i,t,s}, x^{r}_{i,t,s} \ge 0 \qquad \forall i \in I, j \in J, t \in T, m \in M, s \in S, k \in K$$

$$L_{i,j,t,0} = U_{i,j,t,0} = 0, \quad c_{tr_{i,j,0,s}} = 0 \quad \forall i \in I, j \in J, t \in T, s \in S$$

$$m = 0 : \sum_{m=0}^{M} \delta_{i,j,t,m,s} = 1$$

$$Q_{i,j,t,s} \geq \epsilon \left(1 - \delta_{i,j,t,0,s}\right) \quad \forall i, j, t, s, \ \epsilon \text{ is a small positive value close to zero}$$

$$\delta_{i,j,t,m,s} \in \{0,1\} \quad \forall i \in I, j \in J, t \in T, m \in M, s \in S$$

$$\omega_{i,j,k} \in \{0,1\} \quad \forall i \in I, j \in J, k \in K$$

$$\sum_{s=1}^{S} P_s = 1$$

3.4 Formulation of the Risk-Averse Stochastic Programming Model with CVaR

3.4.1 Introduction of the Mean-CVaR Model

In this section, a Mean-CVaR inventory optimization model is developed within the multi-stage stochastic programming framework. Unlike the risk-neutral model that solely minimizes expected cost, this model introduces a risk aversion parameter λ to establish a weighted trade-off between minimizing expected cost and minimizing CVaR. By adjusting the value of λ, the model can simulate the behavior of decision-makers ranging from risk-neutral to highly risk-averse, thereby generating robust inventory policies that are both economically efficient and capable of effectively mitigating extreme risks. Formally, let $\lambda \in [0,1]$ denote the risk aversion parameter and θ the confidence level. The generalized objective function of the Mean-CVaR model can be expressed as a weighted combination of the expected loss cost $\mathbb{E}[C]$ and the conditional value-at-risk $\mathrm{CVaR}_\theta(C)$:

$$\min Z = (1-\lambda) \cdot \mathbb{E}[C] + \lambda \cdot CVaR_\theta(C)$$

where C represents the total system loss cost that varies across random scenarios. To enable solution within a mixed-integer linear programming framework, auxiliary variables are introduced: η to represent the value-at-risk and non-negative auxiliary variables v_s to capture tail losses. Based on the discrete scenario set S, the objective function can be transformed into a linear form:

$$\min Z = (1-\lambda) \sum_{s \in S} P_s C_s + \lambda \left(\eta + \frac{1}{1-\theta} \sum_{s \in S} P_s v_s \right).$$

3.4.2 Formulation of the Multi-Stage Stochastic Inventory Programming Model Incorporating CVaR

This section adopts the Mean-CVaR criterion by incorporating a risk aversion parameter and a CVaR penalty term into the objective function, while augmenting the constraints with the linearized tail loss constraints proposed by Rockafellar and Uryasev [9]. This approach constructs a risk-averse multi-stage stochastic programming inventory model that explicitly accounts for CVaR. The objective function is given as follows:

$$\begin{aligned}\min Z = &\sum_{i=1}^{I}\sum_{j=1}^{J} \alpha_{i,j} X_{i,j} + (1-\lambda)\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{j=1}^{J} P_s * \left(c_{i,j} Q_{i,j,t,s} + \sum_{m=1}^{M} c_{tr_{i,j,m,s}} * \delta_{i,j,t,m,s} \right) \\ &+ (1-\lambda)\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{i=1}^{I} P_s * \left(C_i^{repair} * u^r{}_{i,t-1,s} + h_i * r_{i,t,s} + h_i^u * u^r{}_{i,t,s} + S_i * l_{i,t,s} + C_i^{install} * x^r{}_{i,t,s} \right) . + \lambda \left[\alpha + \frac{1}{1-\theta} \sum_{s=1}^{S} P_s V_s \right]\end{aligned}$$

Constraints: The constraints in this section are identical to those of the risk-neutral multi-stage stochastic programming inventory model and are therefore not repeated here.

$$V_s \geq \sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{j=1}^{J}\left(c_{i,j}Q_{i,j,t,s} + \sum_{m=1}^{M} c_{tr_{i,j,m,s}} * \delta_{i,j,t,m,s}\right) \qquad \forall s \in S$$

$$+\sum_{t=1}^{T}\sum_{i=1}^{I}\left(C_i^{repair} * u^r{}_{i,t-1,s} + h_i * r_{i,t,s} + h_i^u * u^r{}_{i,t,s} + S_i * l_{i,t,s} + C_i^{install} * x^r{}_{i,t,s}\right) - \alpha$$

$$V_s \geq 0, \alpha \in R, s = 1, 2 \cdots S$$

4 Model Solution, Evaluation, and Result Analysis

To improve computational efficiency, this study employs the SCIP solver and implements a tiered cutting plane control strategy: cutting plane iterations are intensified at the root node of the search tree to strengthen the initial lower bound, while iterations are restricted at interior nodes to reduce computational overhead, thereby striking a balance between bound-tightening effectiveness and node processing time. In addition, strong primal heuristics and presolving mechanisms are integrated to further accelerate convergence.

4.1 Solution of the Risk-Neutral Multi-Stage Stochastic Programming Inventory Model

Table 8 Inventory Operation and Maintenance Cost Parameter Settings ($ thousands)

Parameter	**Description**	**Hydraulic Pump LRU**	**Brake LRU**
S_i	Unit shortage penalty cost	12.0	10.0
h_i	Unit holding cost for serviceable inventory	0.25	0.15
h_i^u	Unit holding cost for unserviceable inventory	0.15	0.10
C_i^{repair}	Unit variable repair cost	0.6	0.2
P_i^r	Repair success rate	60%	45%
$C_i^{install}$	Unit installation cost	0.2	0.1

Table 9 Supplier Price and Quality Parameter Settings

Spare Part	**Supplier**	**Defective Rate**	**Delay Rate**
Hydraulic Pump LRU	OEM	1.0%	10.0%
	PMA	5.0%	5.0%
Brake LRU	OEM	1.5%	1.5%
	PMA	5.0%	5.0%

Table 10 Supplier Tiered Pricing and Corresponding Order Quantity Interval Parameters ($ thousands)

Spare Part	**Order Quantity Interval (units)**	**OEM Supplier Unit Price**	**PMA Supplier Unit Price**
Hydraulic Pump LRU	[0, 10]	3.0	2.0
	[11, 30]	2.8	1.8
	[31, 60]	2.6	1.6
Brake LRU	[0, 200]	2.2	1.8
	[201, 400]	2.0	1.6
	[401, 700]	1.7	1.4

Table 11 Transportation Batch Intervals and Fixed Cost Parameters

Spare Part	Transportation Batch	Corresponding Quantity Interval (units)	Fixed Transportation Cost ($ thousands)
Hydraulic Pump LRU	$m = 1$	[0, 3]	3.0
	$m = 2$	[4, 6]	6.0
	$m = 3$	[7, 16]	12.0
	$m = 4$	[17, 30]	24.0
Brake LRU	$m = 1$	[0, 80]	10.0
	$m = 2$	[81, 160]	20.0
	$m = 3$	[161, 240]	30.0
	$m = 4$	[241, 300]	40.0

The multi-stage stochastic programming inventory model is compiled and solved by calling the SCIP solver within a Python environment. The SCIP solver successfully solves the risk-neutral multi-stage stochastic programming model, achieving a computational time of 386 seconds with an optimality gap of less than 0.01%. The optimal ordering strategy and corresponding costs for the risk-neutral multi-stage stochastic programming inventory model are presented in Table 12.

Table 12 Solution Results of the Risk-Neutral Multi-Stage Stochastic Programming Model

Spare Part	Initial Annual Contract	Quarterly Order Quantities				Total Cost ($ thousands)	Gap
	x	Quarter 1	Quarter 2	Quarter 3	Quarter 4		
Hydraulic Pump LRU	22	16	3	2	1	907.13	<0.01%
Brake LRU	320	149	54	77	40		

4.2 Solution of the Risk-Averse Multi-Stage Stochastic Programming Inventory Model with CVaR

Compared to the risk-neutral model, the present model incorporates CVaR, requiring a trade-off between the expected cost and CVaR measure. This increases model scale and algorithmic complexity. With the confidence level set to $\alpha = 0.9$ and the risk aversion parameter set to $\lambda = 0.8$, the SCIP solver completes the solution in 545 seconds, achieving an optimality gap of less than 0.01%. The optimal ordering strategy and corresponding costs for the risk-averse multi-stage stochastic programming inventory model are presented in Table 13.

Table 13 Solution Results of the Risk-Averse Multi-Stage Stochastic Programming Model Incorporating CVaR

Spare Part	Initial Annual Contract	Quarterly Order Quantities				Total Cost ($ thousands)	Gap
	x	Quarter 1	Quarter 2	Quarter 3	Quarter 4		
Hydraulic Pump LRU	25	16	3	3	3	937.36	<0.01%
Brake LRU	332	149	55	75	53		

4.3 Evaluation of Model Adaptability

4.3.1 Adaptability Evaluation of the Risk-Neutral Stochastic Programming Inventory Model

Table 14 Evaluation Metrics for the Stochastic Programming Model ($ thousands)

RP	*EEV*	*WS*	*EVPI*	*VSS*	*REVPI*	*RVSS*
907.13	963.39	864.06	43.07	56.26	4.74%	6.20%

To demonstrate the superiority of the stochastic programming model over traditional deterministic models and to quantify the cost implications of information uncertainty, this study employs the Value of the Stochastic Solution (VSS) and the Expected Value of Perfect Information (EVPI) as evaluation metrics. The obtained VSS and EVPI results are presented as follows.

The computational results demonstrate that the multi-stage stochastic programming model proposed in this study exhibits significant advantages in cost control. The EVPI metric indicates that, if the information barriers arising from uncertainty in the multi-stage environment could be completely eliminated, an additional cost reduction potential of $43,070 could theoretically be unlocked. More critically, the VSS reaches $52,260, a value that directly quantifies the benefit gap between the proposed model and traditional deterministic models, thereby confirming the necessity of adopting stochastic optimization strategies. Given that the inventory model minimizes the objective, the relative metrics RVSS and REVPI further reveal the superior performance of the constructed risk-neutral stochastic programming inventory model.

4.3.2 Adaptability Evaluation of the Stochastic Programming Inventory Model Incorporating CVaR

In the multi-stage model incorporating CVaR developed in this study, the decision objective simultaneously addresses uncertainty fluctuations and risk control. Consequently, directly applying conventional EVPI and VSS metrics is no longer appropriate. To effectively assess the superiority of this risk-averse strategy, matched evaluation metrics—MRVPI and MRVSS—are introduced. These metrics delve deeper into the profit-and-loss distribution information of the EV solution and the WS solution across different random paths. Particularly for a cost-minimization objective, they are more sensitive in capturing and measuring “tail risk” scenarios that lead to substantial expenditures.

With the risk parameters fixed at $\alpha = 0.9$ and $\lambda = 0.8$, the computed evaluation metrics for the multi-stage mean-risk model incorporating CVaR are presented as follows.

Table 15 Evaluation Metrics for the Stochastic Programming Model Incorporating CVaR ($ thousands)

MRRP	*MREV*	*MRWS*	*MREVPI*	*MRVSS*	*REVPI*	*RVSS*
937.36	1172.79	905.85	31.51	235.47	3.36%	25.12%

Based on the above results, we first analyze the Mean-Risk Expected Value of Perfect Information (MREVPI). The computed value is $31,510, with a relative value of 3.36%. This relatively low figure indicates that the multi-stage stochastic programming model constructed in this study, leveraging its multi-stage recourse mechanism, exhibits strong self-correcting capability and robustness. Consequently, its decision outcomes are already very close to the ideal state under perfect information. Next, we examine the more critical Mean-Risk Value of the Stochastic Solution (MRVSS). The results show an MRVSS of $235,470, with a relative value of 25.12%. This substantial value reveals the significant potential losses incurred by deterministic models due to their neglect of fluctuations and tail risks. As indicated by $MRVSS \gg MREVPI$, this strongly demonstrates that, in aviation spare parts inventory management, constructing a risk-averse model that accommodates uncertainty is far more critical than merely pursuing higher accuracy in demand forecasting. It also confirms that the proposed model can significantly reduce generalized loss costs in complex environments while achieving notable advantages in balancing economic efficiency and operational safety.

4.4 Model Robustness and Stability Testing

To verify the reliability of the solution quality of the constructed multi-stage stochastic programming model, this section conducts rigorous testing of both the risk-neutral and risk-averse models using two mechanisms: in-sample testing and out-of-sample testing. The in-sample testing aims to evaluate the fluctuation of objective function values across different scenario generation samples, thereby verifying the stability of the solutions. In contrast, the out-of-sample testing employs a cross-validation approach to examine the adaptability of the fixed strategy in different external environments, thereby assessing the robustness of the model.

4.4.1 In-Sample Stability Testing of the Model

The core logic of in-sample testing is that if the model is stable, the objective function values obtained from different independent scenario sets generated from the same distribution should remain within a relatively small fluctuation range.

(1) In-Sample Testing of the Risk-Neutral Multi-Stage Stochastic Programming Model

Based on the scenario generation framework established earlier, this section sets the size of a single scenario set to 80 scenarios. To perform the stability test, 10 independent test scenario sets of identical size but with randomly distinct realizations are generated using the scenario generation method described in Chapter 2. Subsequently, these 10 scenario sets are individually used as input parameters to solve the risk-neutral model.

Table 16 Objective Values of the Risk-Neutral Inventory Model under Different Scenario Groups

Scenario Group	**Risk-Neutral Objective Value ($ thousands)**	**Scenario Group**	**Risk-Neutral Objective Value ($ thousands)**
Group 1	907.13	Group 6	920.92
Group 2	906.52	Group 7	897.28
Group 3	900.23	Group 8	899.99
Group 4	909.84	Group 9	912.95
Group 5	912.74	Group 10	917.92

According to the statistical results in Table 16, across the 10 different scenario samples, the optimal objective function values of the risk-neutral model range from $897,280 to $920,920. The calculated relative fluctuation amplitude is only 2.6%. This result indicates that the deviations in objective values caused by different sample sets are minimal and well within an acceptable error range. Therefore, it can be concluded that the risk-neutral model passes the in-sample test and exhibits strong solution stability.

(2) In-Sample Testing of the Risk-Averse Multi-Stage Stochastic Programming Model Incorporating CVaR

The same validation procedure is applied to the risk-averse model. The identical 10 independent scenario sets of 80 scenarios each are used as inputs, and the model is solved under the predefined risk parameters. As shown in the experimental data in Table 17 the risk-averse inventory model demonstrates a high degree of stability across the 10 tests, with objective function values ranging from $923,970 to $947,830 and a corresponding relative fluctuation amplitude of 2.5%. This fluctuation level is not only below the conventional 5% threshold but also slightly better than that of the risk-neutral model. These findings fully demonstrate that the risk-averse model incorporating CVaR constructed in this study possesses excellent internal consistency and successfully passes the in-sample stability test.

Table 17 In-Sample Objective Values of the Risk-Averse Inventory Model Incorporating CVaR under Different Scenario Groups

Scenario Group	**Risk-Averse Objective Value ($ thousands)**	**Scenario Group**	**Risk-Averse Objective Value ($ thousands)**
Group 1	937.36	Group 6	943.56
Group 2	934.39	Group 7	925.73
Group 3	923.97	Group 8	930.69
Group 4	942.16	Group 9	945.74
Group 5	944.55	Group 10	947.83

4.4.2 Out-of-Sample Testing of the Model

(1) Out-of-Sample Testing of the Risk-Neutral Stochastic Programming Model

To conduct out-of-sample testing, the second and fifth scenario sets are selected for cross-validation. The risk-neutral model is first solved separately for each of these two scenarios sets to obtain the respective optimal first-stage ordering strategies. Next, the first-stage decision from one set is fixed and applied to the other scenario set, with only the subsequent-stage variables optimized. The total cost under the cross-scenario is then calculated, and the process is repeated in reverse.

Table 18 Decision and Objective Values of the Risk-Neutral Inventory Model under Two Scenario Groups

Scenario Group	Spare Part	Quarterly Order Quantities				Total Cost
		Quarter 1	Quarter 2	Quarter 3	Quarter 4	($ thousands)
Group 2	Hydraulic Pump LRU	14	3	3	0	906.52
	Brake LRU	149	53	75	46	
Group 5	Hydraulic Pump LRU	16	3	4	0	912.74
	Brake LRU	146	55	75	41	

The objective function value when applying the decision from the second set to the fifth scenario set is obtained by computing the objective values for each scenario and taking a weighted sum based on scenario probabilities:

$$Obj_1 = \$949,670$$

Similarly, the objective function value when applying the decision from the fifth set to the second scenario set is computed as:

$$Obj_2 = \$937,390$$

By comparing the original in-sample optimal objective values with those obtained after exchanging decisions, the fluctuation in objective function values is found to be relatively small and within a 10% range. Therefore, the risk-neutral stochastic programming inventory model satisfies the out-of-sample testing criteria.

(2) Out-of-Sample Testing of the Risk-Averse Stochastic Programming Inventory Model Incorporating CVaR

A similar approach to that used in (1) is adopted. The resulting decisions and objective values for the risk-averse inventory model are presented in Table 19.

$$Obj_1 = \$1,030,370$$

$$Obj_2 = \$1,008,540$$

By comparing the original in-sample objective values with those obtained after exchanging decisions, the fluctuation in objective function values is found to be relatively small and within a 10% range. Therefore, the risk-averse stochastic programming inventory model incorporating CVaR satisfies the out-of-sample testing criteria.

Table 19 Decision and Objective Values of the Risk-Averse Inventory Model under Two Scenario Groups

Scenario Group	Spare Part	Quarterly Order Quantities				Total Cost
		Quarter 1	Quarter 2	Quarter 1	Quarter 2	($ thousands)
Group 2	Hydraulic Pump LRU	16	3	1	2	934.39
	Brake LRU	149	53	74	58	
Group 5	Hydraulic Pump LRU	16	6	2	2	944.55
	Brake LRU	146	54	75	55	

5 Conclusions

This study focuses on the challenges in aviation MRO systems, including highly volatile demand, pronounced tail risks, supply delays, and quality uncertainties. It develops a multi-stage stochastic programming inventory optimization framework that integrates scenario tree generation with the Mean-CVaR risk-averse criterion, and validates its effectiveness through numerical experiments. In terms of scenario construction, the proposed approach combines moment matching with entropically regularized optimal transport for scenario generation and reduction. This method preserves key statistical properties and inter-part correlations while improving reduction efficiency and enhancing the representation of extreme tail events, thereby providing reliable inputs for subsequent optimization. With respect to modeling, the incorporation of non-anticipativity constraints enables a genuine multi-stage stochastic programming formulation, allowing decisions to adjust dynamically as information is revealed period by period. This avoids the clairvoyant recourse inherent in traditional two-stage models and better aligns with the actual quarterly rolling decision processes in aviation spare parts procurement and inventory management. Regarding risk control, the Mean-CVaR criterion effectively captures decision-makers' risk-averse preferences, guiding the model to achieve an adjustable trade-off between expected cost and tail risk. This strengthens protection against low-probability, high-impact events and better suits the aviation industry's high safety sensitivity. In summary, compared with deterministic extrapolation methods, the proposed stochastic optimization and risk-averse strategies exhibit greater robustness in uncertain environments, reducing potential losses arising from neglected volatility and tail risks. The framework offers airlines implementable decision support for developing spare parts inventory policies that balance economic efficiency with robustness.

Future research could extend the current work in several directions. Although the model integrates scenario trees and the Mean-CVaR criterion while accounting for demand volatility, inter-part correlations, supply delays, and quality uncertainties, it still lacks explicit representation of how uncertainties evolve dynamically over the operational horizon. In practice, changes in fleet composition, maintenance policies, route networks, and supplier performance continuously affect demand distributions and risk exposure. To address this, future studies could incorporate online learning or rolling update mechanisms to dynamically recalibrate demand parameters and correlation structures, while further refining stochastic lead times and contract compliance constraints. Such enhancements would bring the model closer to real-world decision environments and risk characteristics.

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